

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "6 Hyperbolic functions\6.5 Hyperbolic secant"

Test results for the 16 problems in "6.5.1 (c+d x)^m (a+b sech)^n.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Sech}[a + b x] dx$$

Optimal (type 4, 61 leaves, 5 steps):

$$\frac{2 (c + d x) \operatorname{ArcTan}\left[e^{a+bx}\right]}{b} - \frac{i d \operatorname{PolyLog}\left[2, -i e^{a+bx}\right]}{b^2} + \frac{i d \operatorname{PolyLog}\left[2, i e^{a+bx}\right]}{b^2}$$

Result (type 4, 132 leaves):

$$\frac{1}{2 b^2} \left(4 b c \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} (a + b x)\right]\right] - d (-2 i a + \pi - 2 i b x) (\operatorname{Log}\left[1 - i e^{a+bx}\right] - \operatorname{Log}\left[1 + i e^{a+bx}\right]) + d (-2 i a + \pi) \operatorname{Log}\left[\operatorname{Cot}\left[\frac{1}{4} (2 i a + \pi + 2 i b x)\right]\right] - 2 i d (\operatorname{PolyLog}\left[2, -i e^{a+bx}\right] - \operatorname{PolyLog}\left[2, i e^{a+bx}\right]) \right)$$

Problem 6: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Sech}[a + b x]^2 dx$$

Optimal (type 4, 73 leaves, 5 steps):

$$\frac{(c + d x)^2}{b} - \frac{2 d (c + d x) \operatorname{Log}\left[1 + e^{2(a+bx)}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[2, -e^{2(a+bx)}\right]}{b^3} + \frac{(c + d x)^2 \operatorname{Tanh}[a + b x]}{b}$$

Result (type 4, 277 leaves):

$$\begin{aligned}
& - \frac{2 c d \operatorname{Sech}[a] \left(\operatorname{Cosh}[a] \operatorname{Log}[\operatorname{Cosh}[a] \operatorname{Cosh}[b x] + \operatorname{Sinh}[a] \operatorname{Sinh}[b x]] - b x \operatorname{Sinh}[a] \right)}{b^2 \left(\operatorname{Cosh}[a]^2 - \operatorname{Sinh}[a]^2 \right)} + \\
& \left(d^2 \operatorname{Csch}[a] \left(-b^2 e^{-\operatorname{ArcTanh}[\operatorname{Coth}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[a]^2}} i \operatorname{Coth}[a] \right. \right. \\
& \quad \left. \left. (-b x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[a]]) - \pi \operatorname{Log}[1 + e^{2 b x}] - 2 (i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]]) \operatorname{Log}[1 - e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]])}]] + \right. \right. \\
& \quad \left. \left. \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \operatorname{Log}[i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Coth}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]])}] \right) \right) \operatorname{Sech}[a] \Bigg) / \\
& \left(b^3 \sqrt{\operatorname{Csch}[a]^2 (-\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2)} \right) + \frac{\operatorname{Sech}[a] \operatorname{Sech}[a + b x] \left(c^2 \operatorname{Sinh}[b x] + 2 c d x \operatorname{Sinh}[b x] + d^2 x^2 \operatorname{Sinh}[b x] \right)}{b}
\end{aligned}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Sech}[a + b x]^3 dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{(c + d x) \operatorname{ArcTan}[e^{a + b x}]}{b} - \frac{i d \operatorname{PolyLog}[2, -i e^{a + b x}]}{2 b^2} + \frac{i d \operatorname{PolyLog}[2, i e^{a + b x}]}{2 b^2} + \frac{d \operatorname{Sech}[a + b x]}{2 b^2} + \frac{(c + d x) \operatorname{Sech}[a + b x] \operatorname{Tanh}[a + b x]}{2 b}$$

Result (type 4, 263 leaves):

$$\begin{aligned}
& \frac{c \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]\right]}{b} - \frac{1}{2 b^2} \\
& d \left(\left(-i a + \frac{\pi}{2} - i b x \right) \left(\operatorname{Log}\left[1 - e^{i(-i a + \frac{\pi}{2} - i b x)}\right] - \operatorname{Log}\left[1 + e^{i(-i a + \frac{\pi}{2} - i b x)}\right] \right) - \left(-i a + \frac{\pi}{2} \right) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}\left(-i a + \frac{\pi}{2} - i b x\right)\right]\right] \right) + \\
& \quad i \left(\operatorname{PolyLog}[2, -e^{i(-i a + \frac{\pi}{2} - i b x)}] - \operatorname{PolyLog}[2, e^{i(-i a + \frac{\pi}{2} - i b x)}] \right) + \\
& \frac{d \operatorname{Sech}[a] \operatorname{Sech}[a + b x] \left(\operatorname{Cosh}[a] + b x \operatorname{Sinh}[a] \right)}{2 b^2} + \frac{d x \operatorname{Sech}[a] \operatorname{Sech}[a + b x]^2 \operatorname{Sinh}[b x]}{2 b} + \frac{c \operatorname{Sech}[a + b x] \operatorname{Tanh}[a + b x]}{2 b}
\end{aligned}$$

Problem 12: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Sech}[a + b x]^3}{c + d x} dx$$

Optimal (type 9, 18 leaves, 0 steps):

Unintegrable $\left[\frac{\text{Sech}[a + b x]^3}{c + d x}, x\right]$

Result (type 1, 1 leaves):

???

Test results for the 84 problems in "6.5.2 (e x)^m (a+b sech(c+d x^n))^p.m"

Problem 10: Result more than twice size of optimal antiderivative.

$$\int x^3 (a + b \text{Sech}[c + d x^2])^2 dx$$

Optimal (type 4, 119 leaves, 10 steps):

$$\frac{a^2 x^4}{4} + \frac{2 a b x^2 \text{ArcTan}[e^{c+d x^2}]}{d} - \frac{b^2 \text{Log}[\text{Cosh}[c + d x^2]]}{2 d^2} - \frac{i a b \text{PolyLog}[2, -i e^{c+d x^2}]}{d^2} + \frac{i a b \text{PolyLog}[2, i e^{c+d x^2}]}{d^2} + \frac{b^2 x^2 \text{Tanh}[c + d x^2]}{2 d}$$

Result (type 4, 483 leaves):

$$\frac{x^2 \text{Cosh}[c + d x^2]^2 \text{Sech}[c] (a + b \text{Sech}[c + d x^2])^2 (a^2 d x^2 \text{Cosh}[c] + 2 b^2 \text{Sinh}[c])}{4 d (b + a \text{Cosh}[c + d x^2])^2} - \frac{(b^2 \text{Cosh}[c + d x^2]^2 \text{Sech}[c] (a + b \text{Sech}[c + d x^2])^2 (\text{Cosh}[c] \text{Log}[\text{Cosh}[c] \text{Cosh}[d x^2] + \text{Sinh}[c] \text{Sinh}[d x^2]] - d x^2 \text{Sinh}[c]) - (2 d^2 (b + a \text{Cosh}[c + d x^2])^2 (\text{Cosh}[c]^2 - \text{Sinh}[c]^2)) + \frac{1}{d^2 (b + a \text{Cosh}[c + d x^2])^2} a b \text{Cosh}[c + d x^2]^2 (a + b \text{Sech}[c + d x^2])^2}{\left(-\frac{1}{\sqrt{1 - \text{Coth}[c]^2}} i \text{CsCh}[c] (i (d x^2 + \text{ArcTanh}[\text{Coth}[c]]) (\text{Log}[1 - e^{-d x^2 - \text{ArcTanh}[\text{Coth}[c]}]} - \text{Log}[1 + e^{-d x^2 - \text{ArcTanh}[\text{Coth}[c]}] + i (\text{PolyLog}[2, -e^{-d x^2 - \text{ArcTanh}[\text{Coth}[c]}]} - \text{PolyLog}[2, e^{-d x^2 - \text{ArcTanh}[\text{Coth}[c]}] - \frac{2 \text{ArcTan}\left[\frac{\text{Sinh}[c] + \text{Cosh}[c] \text{Tanh}\left[\frac{d x^2}{2}\right]}{\sqrt{\text{Cosh}[c]^2 - \text{Sinh}[c]^2}}\right] \text{ArcTanh}[\text{Coth}[c]]}{\sqrt{\text{Cosh}[c]^2 - \text{Sinh}[c]^2}} \right) + \frac{b^2 x^2 \text{Cosh}[c + d x^2] \text{Sech}[c] (a + b \text{Sech}[c + d x^2])^2 \text{Sinh}[d x^2]}{2 d (b + a \text{Cosh}[c + d x^2])^2} - \frac{b^2 x^2 \text{Cosh}[c + d x^2]^2 (a + b \text{Sech}[c + d x^2])^2 \text{Tanh}[c]}{2 d (b + a \text{Cosh}[c + d x^2])^2}$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3}{a + b \operatorname{Sech}[c + d x^2]} dx$$

Optimal (type 4, 241 leaves, 11 steps):

$$\frac{x^4}{4 a} - \frac{b x^2 \operatorname{Log}\left[1 + \frac{a e^{c+d x^2}}{b - \sqrt{-a^2 + b^2}}\right]}{2 a \sqrt{-a^2 + b^2} d} + \frac{b x^2 \operatorname{Log}\left[1 + \frac{a e^{-c-d x^2}}{b + \sqrt{-a^2 + b^2}}\right]}{2 a \sqrt{-a^2 + b^2} d} - \frac{b \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^2}}{b - \sqrt{-a^2 + b^2}}\right]}{2 a \sqrt{-a^2 + b^2} d^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{a e^{-c-d x^2}}{b + \sqrt{-a^2 + b^2}}\right]}{2 a \sqrt{-a^2 + b^2} d^2}$$

Result (type 4, 843 leaves):

$$\frac{1}{4 a (a + b \operatorname{Sech}[c + d x^2])} (b + a \operatorname{Cosh}[c + d x^2])$$

$$\left(x^4 + \frac{1}{\sqrt{a^2 - b^2} d^2} 2 b \left(2 (c + d x^2) \operatorname{ArcTan}\left[\frac{(a + b) \operatorname{Coth}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] + 2 \left(c - i \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTan}\left[\frac{(a - b) \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] + \right. \right.$$

$$\left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 \left(\operatorname{ArcTan}\left[\frac{(a + b) \operatorname{Coth}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] + \operatorname{ArcTan}\left[\frac{(a - b) \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2 - b^2} e^{-\frac{c}{2} - \frac{d x^2}{2}}}{\sqrt{2} \sqrt{a} \sqrt{b + a \operatorname{Cosh}[c + d x^2]}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{(a + b) \operatorname{Coth}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] + \operatorname{ArcTan}\left[\frac{(a - b) \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} (c + d x^2)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \operatorname{Cosh}[c + d x^2]}}\right] -$$

$$\left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 \operatorname{ArcTan}\left[\frac{(a - b) \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \operatorname{Log}\left[\frac{(a + b) (-a + b + i \sqrt{a^2 - b^2}) (-1 + \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + i \sqrt{a^2 - b^2} \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}\right] -$$

$$\left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a - b) \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \operatorname{Log}\left[\frac{(a + b) (a - b + i \sqrt{a^2 - b^2}) (1 + \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + i \sqrt{a^2 - b^2} \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}\right] +$$

$$i \left(\operatorname{PolyLog}\left[2, \frac{(b - i \sqrt{a^2 - b^2}) (a + b - i \sqrt{a^2 - b^2} \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + i \sqrt{a^2 - b^2} \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}\right] - \right.$$

$$\left. \operatorname{PolyLog}\left[2, \frac{(b + i \sqrt{a^2 - b^2}) (a + b - i \sqrt{a^2 - b^2} \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + i \sqrt{a^2 - b^2} \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right])}\right] \right) \operatorname{Sech}[c + d x^2]$$

Problem 28: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x (a + b \operatorname{Sech}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x (a + b \operatorname{Sech}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 29: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2 (a + b \operatorname{Sech}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 (a + b \operatorname{Sech}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 30: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^3 (a + b \operatorname{Sech}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^3 (a + b \operatorname{Sech}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 50: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x \left(a + b \operatorname{Sech}[c + d \sqrt{x}] \right)^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x \left(a + b \operatorname{Sech}[c + d \sqrt{x}] \right)^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 51: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2 \left(a + b \operatorname{Sech}[c + d \sqrt{x}] \right)^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 \left(a + b \operatorname{Sech}[c + d \sqrt{x}] \right)^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 75: Unable to integrate problem.

$$\int (e x)^{-1+3 n} \left(a + b \operatorname{Sech}[c + d x^n] \right) dx$$

Optimal (type 4, 217 leaves, 11 steps):

$$\frac{a (e x)^{3 n}}{3 e n} + \frac{2 b x^{-n} (e x)^{3 n} \operatorname{ArcTan}\left[e^{c+d x^n}\right]}{d e n} - \frac{2 i b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -i e^{c+d x^n}\right]}{d^2 e n} +$$

$$\frac{2 i b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, i e^{c+d x^n}\right]}{d^2 e n} + \frac{2 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -i e^{c+d x^n}\right]}{d^3 e n} - \frac{2 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, i e^{c+d x^n}\right]}{d^3 e n}$$

Result (type 8, 24 leaves):

$$\int (e x)^{-1+3 n} (a + b \operatorname{Sech}[c + d x^n]) dx$$

Problem 78: Unable to integrate problem.

$$\int (e x)^{-1+3 n} (a + b \operatorname{Sech}[c + d x^n])^2 dx$$

Optimal (type 4, 363 leaves, 16 steps):

$$\frac{a^2 (e x)^{3 n}}{3 e n} + \frac{b^2 x^{-n} (e x)^{3 n}}{d e n} + \frac{4 a b x^{-n} (e x)^{3 n} \operatorname{ArcTan}\left[e^{c+d x^n}\right]}{d e n} - \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + e^{2(c+d x^n)}\right]}{d^2 e n} - \frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -i e^{c+d x^n}\right]}{d^2 e n} + \frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, i e^{c+d x^n}\right]}{d^2 e n} - \frac{b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -e^{2(c+d x^n)}\right]}{d^3 e n} + \frac{4 i a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -i e^{c+d x^n}\right]}{d^3 e n} - \frac{4 i a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, i e^{c+d x^n}\right]}{d^3 e n} + \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Tanh}[c + d x^n]}{d e n}$$

Result (type 8, 26 leaves):

$$\int (e x)^{-1+3 n} (a + b \operatorname{Sech}[c + d x^n])^2 dx$$

Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{a + b \operatorname{Sech}[c + d x^n]} dx$$

Optimal (type 4, 307 leaves, 12 steps):

$$\frac{(e x)^{2 n}}{2 a e n} - \frac{b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} d e n} + \frac{b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} d e n} - \frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} d^2 e n} + \frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} d^2 e n}$$

Result (type 4, 859 leaves):

$$\begin{aligned}
& \frac{1}{2 a e n (a+b \operatorname{Sech}[c+d x^n])} (e x)^{2 n} (b+a \operatorname{Cosh}[c+d x^n]) \\
& \left(1 + \frac{1}{\sqrt{a^2-b^2} d^2} 2 b x^{-2 n} \left(2 (c+d x^n) \operatorname{ArcTan}\left[\frac{(a+b) \operatorname{Coth}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] + 2 \left(c-i \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] + \right. \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 \left(\operatorname{ArcTan}\left[\frac{(a+b) \operatorname{Coth}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] + \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{c+d x^n}{2}}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x^n]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{(a+b) \operatorname{Coth}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] + \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2}(c+d x^n)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x^n]}}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) (-a+b+i \sqrt{a^2-b^2}) (-1+\operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}{a (a+b+i \sqrt{a^2-b^2} \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) (a-b+i \sqrt{a^2-b^2}) (1+\operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}{a (a+b+i \sqrt{a^2-b^2} \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(b-i \sqrt{a^2-b^2}) (a+b-i \sqrt{a^2-b^2} \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}{a (a+b+i \sqrt{a^2-b^2} \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(b+i \sqrt{a^2-b^2}) (a+b-i \sqrt{a^2-b^2} \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}{a (a+b+i \sqrt{a^2-b^2} \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right])}\right] \right) \operatorname{Sech}[c+d x^n]
\end{aligned}$$

Problem 81: Unable to integrate problem.

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Sech}[c+d x^n]} dx$$

Optimal (type 4, 452 leaves, 14 steps):

$$\begin{aligned}
& \frac{(e x)^{3 n}}{3 a e n} - \frac{b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} + \frac{b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} - \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} + \\
& \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} + \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^3 e n} - \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^3 e n}
\end{aligned}$$

Result (type 8, 26 leaves):

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Sech}[c+d x^n]} dx$$

Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{(a+b \operatorname{Sech}[c+d x^n])^2} dx$$

Optimal (type 4, 717 leaves, 23 steps):

$$\begin{aligned} & \frac{(e x)^{2 n}}{2 a^2 e n} + \frac{b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d e n} - \frac{2 b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d e n} - \\ & \frac{b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d e n} + \frac{2 b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d e n} - \frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[b+a \operatorname{Cosh}[c+d x^n]]}{a^2 (a^2-b^2) d^2 e n} + \\ & \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d^2 e n} - \frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d^2 e n} - \\ & \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d^2 e n} + \frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d^2 e n} + \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Sinh}[c+d x^n]}{a (a^2-b^2) d e n (b+a \operatorname{Cosh}[c+d x^n])} \end{aligned}$$

Result (type 4, 2651 leaves):

$$\begin{aligned} & \frac{1}{(a^2-b^2)^{3/2} d^2 n (a+b \operatorname{Sech}[c+d x^n])^2} 2 b x^{1-2 n} (e x)^{-1+2 n} (b+a \operatorname{Cosh}[c+d x^n])^2 \\ & \left(2 (i c + i d x^n) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(i c + i d x^n)\right]}{\sqrt{a^2-b^2}}\right] - 2 \left(i c + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(i c + i d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) + \\ & \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(i c + i d x^n)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(i c + i d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2} i (i c + i d x^n)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x^n]}}\right] + \\ & \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(i c + i d x^n)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(i c + i d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2} i (i c + i d x^n)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x^n]}}\right] - \\ & \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(i c + i d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b-i \sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(i c + i d x^n)\right])}{a (a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(i c + i d x^n)\right])}\right] + \end{aligned}$$

$$\begin{aligned}
& \left(-\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \text{Log}\left[1 - \frac{(b+i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}{a(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(b-i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}{a(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}\right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{(b+i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}{a(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}\right] \right) \text{Sech}[c+dx^n]^2 - \\
& \frac{1}{a^2(a^2-b^2)^{3/2} d^n (a+b \text{Sech}[c+dx^n])^2} b^3 x^{1-2n} (ex)^{-1+2n} (b+a \text{Cosh}[c+dx^n])^2 \\
& \left(2(ic+id x^n) \text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] - 2\left(ic+\text{ArcCos}\left[-\frac{b}{a}\right]\right) \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] + \right. \\
& \left(\text{ArcCos}\left[-\frac{b}{a}\right] - 2i \left(\text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] - \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2}i(ic+id x^n)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \text{Cosh}[c+dx^n]}}\right] + \\
& \left(\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \left(\text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] - \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2}i(ic+id x^n)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \text{Cosh}[c+dx^n]}}\right] - \\
& \left(\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \text{Log}\left[1 - \frac{(b-i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}{a(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}\right] + \\
& \left(-\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(ic+id x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \text{Log}\left[1 - \frac{(b+i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}{a(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(b-i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}{a(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}\right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{(b+i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}{a(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(ic+id x^n)\right])}\right] \right) \text{Sech}[c+dx^n]^2 + \\
& \left(x^{1-n} (ex)^{-1+2n} (b+a \text{Cosh}[c+dx^n])^2 \text{Sech}[c] \text{Sech}[c+dx^n]^2 (a^2 dx^n \text{Cosh}[c] - b^2 dx^n \text{Cosh}[c] + 2b^2 \text{Sinh}[c]) \right) / \\
& \left(2a^2(a-b)(a+b) d^n (a+b \text{Sech}[c+dx^n])^2 \right) -
\end{aligned}$$

$$\left(\begin{array}{l}
b^2 x^{1-2n} (e x)^{-1+2n} (b + a \operatorname{Cosh}[c + d x^n])^2 \operatorname{Sech}[c] \operatorname{Sech}[c + d x^n]^2 \\
\left(a \operatorname{Cosh}[c] \operatorname{Log}[b + a \operatorname{Cosh}[c] \operatorname{Cosh}[d x^n] + a \operatorname{Sinh}[c] \operatorname{Sinh}[d x^n]] - a d x^n \operatorname{Sinh}[c] + \frac{2 a b \operatorname{ArcTan}\left[\frac{a \operatorname{Sinh}[c] + (-b + a \operatorname{Cosh}[c]) \operatorname{Tanh}\left[\frac{d x^n}{2}\right]}{\sqrt{-b^2 + a^2 \operatorname{Cosh}[c]^2 - a^2 \operatorname{Sinh}[c]^2}}\right] \operatorname{Sinh}[c]}{\sqrt{-b^2 + a^2 \operatorname{Cosh}[c]^2 - a^2 \operatorname{Sinh}[c]^2}} \right) \operatorname{Sinh}[c] \right) \\
\left(a (a^2 - b^2) d^2 n (a + b \operatorname{Sech}[c + d x^n])^2 (a^2 \operatorname{Cosh}[c]^2 - a^2 \operatorname{Sinh}[c]^2) \right) + \\
\frac{b^2 x^{1-n} (e x)^{-1+2n} (b + a \operatorname{Cosh}[c + d x^n]) \operatorname{Sech}[c] \operatorname{Sech}[c + d x^n]^2 (b \operatorname{Sinh}[c] - a \operatorname{Sinh}[d x^n])}{a^2 (-a + b) (a + b) d n (a + b \operatorname{Sech}[c + d x^n])^2} + \\
\frac{b^2 x^{1-n} (e x)^{-1+2n} (b + a \operatorname{Cosh}[c + d x^n])^2 \operatorname{Sech}[c + d x^n]^2 \operatorname{Tanh}[c]}{a^2 (-a^2 + b^2) d n (a + b \operatorname{Sech}[c + d x^n])^2} - \\
\frac{2 b^3 x^{1-2n} (e x)^{-1+2n} \operatorname{ArcTan}\left[\frac{(-a+b) \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] (b + a \operatorname{Cosh}[c + d x^n])^2 \operatorname{Sech}[c + d x^n]^2 \operatorname{Tanh}[c]}{a^2 (a^2 - b^2)^{3/2} d^2 n (a + b \operatorname{Sech}[c + d x^n])^2}
\end{array} \right) /$$

Problem 84: Attempted integration timed out after 120 seconds.

$$\int \frac{(e x)^{-1+3n}}{(a + b \operatorname{Sech}[c + d x^n])^2} dx$$

Optimal (type 4, 1284 leaves, 32 steps):

$$\begin{aligned}
& \frac{(e x)^{3 n}}{3 a^2 e n} + \frac{b^2 x^{-n} (e x)^{3 n}}{a^2 (a^2 - b^2) d e n} - \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} + \frac{b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} \\
& - \frac{2 b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} - \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} - \frac{b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} + \\
& - \frac{2 b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} - \frac{2 b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} + \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} - \\
& - \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} - \frac{2 b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} - \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} + \\
& - \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} - \frac{2 b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} + \frac{4 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} + \\
& - \frac{2 b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} - \frac{4 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} + \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Sinh}[c + d x^n]}{a (a^2 - b^2) d e n (b + a \operatorname{Cosh}[c + d x^n])}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Test results for the 201 problems in "6.5.3 Hyperbolic secant functions.m"

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\operatorname{Sech}[a + b x]^2} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{\operatorname{ArcSin}[\operatorname{Tanh}[a + b x]]}{b}$$

Result (type 3, 34 leaves):

$$\frac{2 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Cosh}[a+bx] \sqrt{\operatorname{Sech}[a+bx]^2}}{b}$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int (a + a \operatorname{Sech}[c+dx])^{3/2} dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Tanh}[c+dx]}{\sqrt{a+a \operatorname{Sech}[c+dx]}}\right]}{d} + \frac{2 a^2 \operatorname{Tanh}[c+dx]}{d \sqrt{a+a \operatorname{Sech}[c+dx]}}$$

Result (type 3, 135 leaves):

$$\frac{1}{d(1+e^{c+dx})} a \left(-2 + 2 e^{c+dx} + c \sqrt{1+e^{2(c+dx)}} + d \sqrt{1+e^{2(c+dx)}} x + \sqrt{1+e^{2(c+dx)}} \operatorname{ArcSinh}\left[e^{c+dx}\right] - \sqrt{1+e^{2(c+dx)}} \operatorname{Log}\left[1+\sqrt{1+e^{2(c+dx)}}\right] \right) \sqrt{a(1+\operatorname{Sech}[c+dx])}$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+a \operatorname{Sech}[c+dx]} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Tanh}[c+dx]}{\sqrt{a+a \operatorname{Sech}[c+dx]}}\right]}{d}$$

Result (type 3, 77 leaves):

$$\frac{\sqrt{1+e^{2(c+dx)}} \left(c+dx + \operatorname{ArcSinh}\left[e^{c+dx}\right] - \operatorname{Log}\left[1+\sqrt{1+e^{2(c+dx)}}\right] \right) \sqrt{a(1+\operatorname{Sech}[c+dx])}}{d(1+e^{c+dx})}$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+a \operatorname{Sech}[c+dx])^{3/2}} dx$$

Optimal (type 3, 114 leaves, 6 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Tanh}[c+dx]}{\sqrt{a+a \operatorname{Sech}[c+dx]}}\right]}{a^{3/2} d} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Tanh}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sech}[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\operatorname{Tanh}[c+dx]}{2 d (a+a \operatorname{Sech}[c+dx])^{3/2}}$$

Result (type 3, 231 leaves):

$$\left(\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sech}[c+dx]^{3/2} \right. \\ \left. \left(\sqrt{2} e^{\frac{1}{2}(c-dx)} \sqrt{\frac{e^{c+dx}}{1+e^{2(c+dx)}}} \sqrt{1+e^{2(c+dx)}} \left(4c+4dx+4 \operatorname{ArcSinh}[e^{c+dx}] - 5\sqrt{2} \operatorname{Log}[1+e^{c+dx}] - 4 \operatorname{Log}[1+\sqrt{1+e^{2(c+dx)}}] + \right. \right. \right. \\ \left. \left. \left. 5\sqrt{2} \operatorname{Log}[1-e^{c+dx}+\sqrt{2}\sqrt{1+e^{2(c+dx)}}] \right) - \frac{2 \operatorname{Sech}\left[\frac{1}{2}(c+dx)\right] \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\operatorname{Sech}[c+dx]}} \right) \right) / \left(2 d (a (1 + \operatorname{Sech}[c+dx]))^{3/2} \right)$$

Problem 85: Result more than twice size of optimal antiderivative.

$$\int \sqrt{3+3 \operatorname{Sech}[x]} dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$2 \sqrt{3} \operatorname{ArcTanh}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{1+\operatorname{Sech}[x]}}\right]$$

Result (type 3, 54 leaves):

$$\frac{\sqrt{3} \sqrt{1+e^{2x}} \left(x + \operatorname{ArcSinh}[e^x] - \operatorname{Log}[1+\sqrt{1+e^{2x}}] \right) \sqrt{1+\operatorname{Sech}[x]}}{1+e^x}$$

Problem 86: Result more than twice size of optimal antiderivative.

$$\int \sqrt{3-3 \operatorname{Sech}[x]} dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$2 \sqrt{3} \operatorname{ArcTanh}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{1-\operatorname{Sech}[x]}}\right]$$

Result (type 3, 56 leaves):

$$\frac{\sqrt{3} \sqrt{1 + e^{2x}} \left(-x + \text{ArcSinh}[e^x] + \text{Log}\left[1 + \sqrt{1 + e^{2x}}\right] \right) \sqrt{1 - \text{Sech}[x]}}{-1 + e^x}$$

Problem 94: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + b \text{Sech}[c + d x]}} dx$$

Optimal (type 4, 106 leaves, 1 step):

$$\frac{2 \sqrt{a + b} \text{Coth}[c + d x] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sech}[c+d x])}{a-b}}}{a d}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a + b \text{Sech}[c + d x]}} dx$$

Problem 129: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[c + d x]^3 \sqrt{a + b \text{Sech}[c + d x]} dx$$

Optimal (type 3, 217 leaves, 13 steps):

$$\frac{2 \sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a}}\right]}{d} - \frac{a \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{\sqrt{a-b} d} + \frac{3 b \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{4 \sqrt{a-b} d} - \frac{a \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{\sqrt{a+b} d} - \frac{3 b \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{4 \sqrt{a+b} d} - \frac{\text{Coth}[c + d x]^2 \sqrt{a + b \text{Sech}[c + d x]}}{2 d}$$

Result (type 3, 844 leaves):

$$\begin{aligned}
& \frac{\left(-\frac{1}{2} - \frac{1}{2} \operatorname{Csch}[c + dx]^2\right) \sqrt{a + b \operatorname{Sech}[c + dx]}}{d} + \frac{1}{4d \sqrt{b + a \operatorname{Cosh}[c + dx]} \sqrt{\operatorname{Sech}[c + dx]}} \\
& \left(\left(3b \left(\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+dx]}}{\sqrt{-a-b} \sqrt{a \operatorname{Cosh}[c+dx]}}\right] + \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+dx]}}{\sqrt{a-b} \sqrt{a \operatorname{Cosh}[c+dx]}}\right] \right) \sqrt{\frac{-a+a \operatorname{Cosh}[c+dx]}{a+a \operatorname{Cosh}[c+dx]}} (a+a \operatorname{Cosh}[c+dx]) \right) / \right. \\
& \quad \left. \left(\sqrt{a} \sqrt{-a-b} \sqrt{a-b} \sqrt{-1+\operatorname{Cosh}[c+dx]} \sqrt{a \operatorname{Cosh}[c+dx]} \sqrt{1+\operatorname{Cosh}[c+dx]} \sqrt{\operatorname{Sech}[c+dx]} \right) + \right. \\
& \quad \left. \left(2 \left(\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+dx]}}{\sqrt{-a-b} \sqrt{a \operatorname{Cosh}[c+dx]}}\right] - \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+dx]}}{\sqrt{a-b} \sqrt{a \operatorname{Cosh}[c+dx]}}\right] \right) \sqrt{a \operatorname{Cosh}[c+dx]} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{-a+a \operatorname{Cosh}[c+dx]}{a+a \operatorname{Cosh}[c+dx]}} (a+a \operatorname{Cosh}[c+dx]) \sqrt{\operatorname{Sech}[c+dx]} \right) / \left(\sqrt{a} \sqrt{-a-b} \sqrt{a-b} \sqrt{-1+\operatorname{Cosh}[c+dx]} \sqrt{1+\operatorname{Cosh}[c+dx]} \right) + \right. \\
& \quad \left. \left(2a \left(\sqrt{-a-b} \left(-4\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{b+a \operatorname{Cosh}[c+dx]}}{\sqrt{-a \operatorname{Cosh}[c+dx]}}\right] + \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+dx]}}{\sqrt{a-b} \sqrt{-a \operatorname{Cosh}[c+dx]}}\right] \right) \right. \right. \\
& \quad \left. \left. \sqrt{a} \sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+dx]}}{\sqrt{-a-b} \sqrt{-a \operatorname{Cosh}[c+dx]}}\right] \right) \sqrt{-a \operatorname{Cosh}[c+dx]} \sqrt{\frac{-a+a \operatorname{Cosh}[c+dx]}{a+a \operatorname{Cosh}[c+dx]}} \right. \\
& \quad \left. \left. (a+a \operatorname{Cosh}[c+dx]) \operatorname{Cosh}[2(c+dx)] \sqrt{\operatorname{Sech}[c+dx]} \right) / \left(\sqrt{-a-b} \sqrt{a-b} \sqrt{-1+\operatorname{Cosh}[c+dx]} \sqrt{1+\operatorname{Cosh}[c+dx]} \right. \right. \\
& \quad \left. \left. \left(a^2 - 2b^2 + 4b(b+a \operatorname{Cosh}[c+dx]) - 2(b+a \operatorname{Cosh}[c+dx])^2 \right) \right) \right) \sqrt{a+b \operatorname{Sech}[c+dx]}
\end{aligned}$$

Problem 130: Attempted integration timed out after 120 seconds.

$$\int \sqrt{a + b \operatorname{Sech}[c + dx]} \operatorname{Tanh}[c + dx]^2 dx$$

Optimal (type 4, 344 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{3b^2d} 2a(a-b)\sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}} - \frac{1}{3bd} \\
& 2\sqrt{a+b}(a+2b)\operatorname{Coth}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}} + \\
& \frac{2\sqrt{a+b}\operatorname{Coth}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}}}{d} - \frac{2\sqrt{a+b}\operatorname{Sech}[c+dx] \operatorname{Tanh}[c+dx]}{3d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 131: Unable to integrate problem.

$$\int \sqrt{a+b\operatorname{Sech}[c+dx]} dx$$

Optimal (type 4, 125 leaves, 1 step):

$$\begin{aligned}
& \frac{1}{\sqrt{a+b}d} 2\operatorname{Coth}[c+dx] \operatorname{EllipticPi}\left[\frac{a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}}{\sqrt{a+b}\operatorname{Sech}[c+dx]}\right], \frac{a-b}{a+b}\right] \\
& \sqrt{-\frac{b(1-\operatorname{Sech}[c+dx])}{a+b\operatorname{Sech}[c+dx]}} \sqrt{\frac{b(1+\operatorname{Sech}[c+dx])}{a+b\operatorname{Sech}[c+dx]}} (a+b\operatorname{Sech}[c+dx])
\end{aligned}$$

Result (type 8, 16 leaves):

$$\int \sqrt{a+b\operatorname{Sech}[c+dx]} dx$$

Problem 132: Attempted integration timed out after 120 seconds.

$$\int \operatorname{Coth}[c+dx]^2 \sqrt{a+b\operatorname{Sech}[c+dx]} dx$$

Optimal (type 4, 246 leaves, 5 steps):

$$\frac{\sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sech}[c+dx])}{a-b}}}{d} - \frac{\operatorname{Coth}[c+dx] \sqrt{a+b} \operatorname{Sech}[c+dx]}{d} + \frac{1}{\sqrt{a+b} d}$$

$$2 \operatorname{Coth}[c+dx] \operatorname{EllipticPi}\left[\frac{a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}}{\sqrt{a+b} \operatorname{Sech}[c+dx]}\right], \frac{a-b}{a+b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b} \operatorname{Sech}[c+dx]} \sqrt{\frac{b(1+\operatorname{Sech}[c+dx])}{a+b} \operatorname{Sech}[c+dx]} (a+b \operatorname{Sech}[c+dx])$$

Result (type 1, 1 leaves):

???

Problem 135: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+dx]}{\sqrt{a+b} \operatorname{Sech}[c+dx]} dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+dx]}{\sqrt{a}}\right]}{\sqrt{a} d}$$

Result (type 3, 82 leaves):

$$\frac{2 \sqrt{b+a} \operatorname{Cosh}[c+dx] \operatorname{Log}\left[a \sqrt{b+a} \operatorname{Cosh}[c+dx] + \frac{a^{3/2}}{\sqrt{\operatorname{Sech}[c+dx]}}\right] \sqrt{\operatorname{Sech}[c+dx]}}{\sqrt{a} d \sqrt{a+b} \operatorname{Sech}[c+dx]}$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c+dx]}{\sqrt{a+b} \operatorname{Sech}[c+dx]} dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+dx]}{\sqrt{a}}\right]}{\sqrt{a} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+dx]}{\sqrt{a-b}}\right]}{\sqrt{a-b} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right]}{\sqrt{a+b} d}$$

Result (type 3, 419 leaves):

$$\frac{1}{2 a \sqrt{-a-b} \sqrt{a-b} d \sqrt{a+b} \operatorname{Sech}[c+d x]} \sqrt{b+a} \operatorname{Cosh}[c+d x]$$

$$\left(4 \sqrt{-a-b} \sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{b+a} \operatorname{Cosh}[c+d x]}{\sqrt{-a} \operatorname{Cosh}[c+d x]}\right] \sqrt{-a} \operatorname{Cosh}[c+d x] - \sqrt{a} \sqrt{-a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a} \operatorname{Cosh}[c+d x]}{\sqrt{a-b} \sqrt{-a} \operatorname{Cosh}[c+d x]}\right] \sqrt{-a} \operatorname{Cosh}[c+d x] + \right.$$

$$\left. \sqrt{a} \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{b+a} \operatorname{Cosh}[c+d x]}{\sqrt{-a-b} \sqrt{-a} \operatorname{Cosh}[c+d x]}\right] \sqrt{-a} \operatorname{Cosh}[c+d x] + \sqrt{a} \sqrt{-a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a} \operatorname{Cosh}[c+d x]}{\sqrt{-a-b} \sqrt{a} \operatorname{Cosh}[c+d x]}\right] \sqrt{a} \operatorname{Cosh}[c+d x] - \right.$$

$$\left. \sqrt{a} \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{b+a} \operatorname{Cosh}[c+d x]}{\sqrt{a-b} \sqrt{a} \operatorname{Cosh}[c+d x]}\right] \sqrt{a} \operatorname{Cosh}[c+d x] \right) \operatorname{Sech}[c+d x]$$

Problem 137: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c+d x]^3}{\sqrt{a+b} \operatorname{Sech}[c+d x]} dx$$

Optimal (type 3, 262 leaves, 11 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a}}\right]}{\sqrt{a} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a-b}}\right]}{\sqrt{a-b} d} + \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a-b}}\right]}{4 (a-b)^{3/2} d} -$$

$$\frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a+b}}\right]}{4 (a+b)^{3/2} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a+b}}\right]}{\sqrt{a+b} d} - \frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{4 (a+b) d (1-\operatorname{Sech}[c+d x])} - \frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{4 (a-b) d (1+\operatorname{Sech}[c+d x])}$$

Result (type 3, 925 leaves):

$$\begin{aligned}
& \frac{1}{4(a-b)(a+b)d\sqrt{a+b}\operatorname{Sech}[c+dx]} \\
& \sqrt{b+a\operatorname{Cosh}[c+dx]} \left(\left(\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{b+a\operatorname{Cosh}[c+dx]}}{\sqrt{-a-b}\sqrt{a\operatorname{Cosh}[c+dx]}}\right] + \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{b+a\operatorname{Cosh}[c+dx]}}{\sqrt{a-b}\sqrt{a\operatorname{Cosh}[c+dx]}}\right] \right) \sqrt{\frac{-a+a\operatorname{Cosh}[c+dx]}{a+a\operatorname{Cosh}[c+dx]}} \right. \\
& \left. (a+a\operatorname{Cosh}[c+dx]) \right) / \left(\sqrt{-a-b}\sqrt{a-b}\sqrt{-1+\operatorname{Cosh}[c+dx]}\sqrt{a\operatorname{Cosh}[c+dx]}\sqrt{1+\operatorname{Cosh}[c+dx]}\sqrt{\operatorname{Sech}[c+dx]} \right) + \\
& \left((2a^2-3b^2) \left(\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{b+a\operatorname{Cosh}[c+dx]}}{\sqrt{-a-b}\sqrt{a\operatorname{Cosh}[c+dx]}}\right] - \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{b+a\operatorname{Cosh}[c+dx]}}{\sqrt{a-b}\sqrt{a\operatorname{Cosh}[c+dx]}}\right] \right) \sqrt{a\operatorname{Cosh}[c+dx]} \right. \\
& \left. \sqrt{\frac{-a+a\operatorname{Cosh}[c+dx]}{a+a\operatorname{Cosh}[c+dx]}} (a+a\operatorname{Cosh}[c+dx]) \sqrt{\operatorname{Sech}[c+dx]} \right) / \left(a^{3/2}\sqrt{-a-b}\sqrt{a-b}\sqrt{-1+\operatorname{Cosh}[c+dx]}\sqrt{1+\operatorname{Cosh}[c+dx]} \right) + \\
& \left((2a^2-2b^2) \left(\sqrt{-a-b} \left(-4\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{b+a\operatorname{Cosh}[c+dx]}}{\sqrt{-a}\operatorname{Cosh}[c+dx]}}\right] + \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{b+a\operatorname{Cosh}[c+dx]}}{\sqrt{a-b}\sqrt{-a}\operatorname{Cosh}[c+dx]}}\right] - \sqrt{a}\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{b+a\operatorname{Cosh}[c+dx]}}{\sqrt{-a-b}\sqrt{-a}\operatorname{Cosh}[c+dx]}}\right] \right) \sqrt{-a}\operatorname{Cosh}[c+dx] \sqrt{\frac{-a+a\operatorname{Cosh}[c+dx]}{a+a\operatorname{Cosh}[c+dx]}} (a+a\operatorname{Cosh}[c+dx]) \operatorname{Cosh}[2(c+dx)] \sqrt{\operatorname{Sech}[c+dx]} \right) / \\
& \left(\sqrt{-a-b}\sqrt{a-b}\sqrt{-1+\operatorname{Cosh}[c+dx]}\sqrt{1+\operatorname{Cosh}[c+dx]} \left(a^2-2b^2+4b(b+a\operatorname{Cosh}[c+dx]) - 2(b+a\operatorname{Cosh}[c+dx])^2 \right) \right) \\
& \sqrt{\operatorname{Sech}[c+dx]} + \frac{(b+a\operatorname{Cosh}[c+dx]) \left(-\frac{a}{2(a^2-b^2)} + \frac{(a-b)\operatorname{Cosh}[c+dx]\operatorname{Csch}[c+dx]^2}{2(-a^2+b^2)} \right) \operatorname{Sech}[c+dx]}{d\sqrt{a+b}\operatorname{Sech}[c+dx]}
\end{aligned}$$

Problem 138: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Tanh}[c+dx]^4}{\sqrt{a+b}\operatorname{Sech}[c+dx]} dx$$

Optimal (type 4, 610 leaves, 11 steps):

$$\begin{aligned}
& -\frac{1}{b^2 d} 4 (a-b) \sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}} + \frac{1}{15 b^4 d} \\
& 2 (a-b) \sqrt{a+b} (8 a^2+9 b^2) \operatorname{Coth}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}} - \\
& \frac{4 \sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}}}{b d} + \frac{1}{15 b^3 d} \\
& 2 \sqrt{a+b} (8 a^2-2 a b+9 b^2) \operatorname{Coth}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}} + \\
& \frac{2 \sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}}}{a d} - \\
& \frac{8 a \sqrt{a+b \operatorname{Sech}[c+dx]} \operatorname{Tanh}[c+dx]}{15 b^2 d} + \frac{2 \operatorname{Sech}[c+dx] \sqrt{a+b \operatorname{Sech}[c+dx]} \operatorname{Tanh}[c+dx]}{5 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 139: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Tanh}[c+dx]^2}{\sqrt{a+b \operatorname{Sech}[c+dx]}} dx$$

Optimal (type 4, 310 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{b^2 d} 2 (a-b) \sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}} - \\
& \frac{2 \sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}}}{b d} + \\
& \frac{2 \sqrt{a+b} \operatorname{Coth}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+dx])}{a-b}}}{a d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 140: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + b \operatorname{Sech}[c + d x]}} dx$$

Optimal (type 4, 106 leaves, 1 step):

$$\frac{2 \sqrt{a + b} \operatorname{Coth}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{a d}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a + b \operatorname{Sech}[c + d x]}} dx$$

Problem 141: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Coth}[c + d x]^2}{\sqrt{a + b \operatorname{Sech}[c + d x]}} dx$$

Optimal (type 4, 362 leaves, 9 steps):

$$\frac{\operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{\sqrt{a + b} d} -$$

$$\frac{\operatorname{Coth}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{\sqrt{a + b} d} +$$

$$\frac{2 \sqrt{a + b} \operatorname{Coth}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{a d} -$$

$$\frac{\operatorname{Coth}[c + d x]}{d \sqrt{a + b \operatorname{Sech}[c + d x]}} - \frac{b^2 \operatorname{Tanh}[c + d x]}{(a^2 - b^2) d \sqrt{a + b \operatorname{Sech}[c + d x]}}$$

Result (type 1, 1 leaves):

???

Problem 145: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + d x]}{(a + b \operatorname{Sech}[c + d x])^{3/2}} dx$$

Optimal (type 3, 142 leaves, 7 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{(a-b)^{3/2} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{(a+b)^{3/2} d} + \frac{2 b^2}{a (a^2 - b^2) d \sqrt{a + b \operatorname{Sech}[c + d x]}}$$

Result (type 3, 927 leaves):

$$\begin{aligned}
& - \frac{1}{2 a (-a + b) (a + b) d (a + b \operatorname{Sech}[c + d x])^{3/2}} (b + a \operatorname{Cosh}[c + d x])^{3/2} \\
& \left(- \left(\left(2 \sqrt{a} b \left(\sqrt{a-b} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c + d x]}}{\sqrt{-a-b} \sqrt{a \operatorname{Cosh}[c + d x]}} \right] + \sqrt{-a-b} \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c + d x]}}{\sqrt{a-b} \sqrt{a \operatorname{Cosh}[c + d x]}} \right] \right) \sqrt{\frac{-a + a \operatorname{Cosh}[c + d x]}{a + a \operatorname{Cosh}[c + d x]}} \right. \right. \\
& \left. \left. (a + a \operatorname{Cosh}[c + d x]) \right) / \left(\sqrt{-a-b} \sqrt{a-b} \sqrt{-1 + \operatorname{Cosh}[c + d x]} \sqrt{a \operatorname{Cosh}[c + d x]} \sqrt{1 + \operatorname{Cosh}[c + d x]} \sqrt{\operatorname{Sech}[c + d x]} \right) \right) + \\
& \left((a^2 + b^2) \left(\sqrt{a-b} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c + d x]}}{\sqrt{-a-b} \sqrt{a \operatorname{Cosh}[c + d x]}} \right] - \sqrt{-a-b} \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c + d x]}}{\sqrt{a-b} \sqrt{a \operatorname{Cosh}[c + d x]}} \right] \right) \sqrt{a \operatorname{Cosh}[c + d x]} \right. \\
& \left. \sqrt{\frac{-a + a \operatorname{Cosh}[c + d x]}{a + a \operatorname{Cosh}[c + d x]}} (a + a \operatorname{Cosh}[c + d x]) \sqrt{\operatorname{Sech}[c + d x]} \right) / \left(a^{3/2} \sqrt{-a-b} \sqrt{a-b} \sqrt{-1 + \operatorname{Cosh}[c + d x]} \sqrt{1 + \operatorname{Cosh}[c + d x]} \right) + \\
& \left((a^2 - b^2) \left(\sqrt{-a-b} \left(-4 \sqrt{a-b} \operatorname{ArcTan} \left[\frac{\sqrt{b+a \operatorname{Cosh}[c + d x]}}{\sqrt{-a \operatorname{Cosh}[c + d x]}} \right] + \sqrt{a} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c + d x]}}{\sqrt{a-b} \sqrt{-a \operatorname{Cosh}[c + d x]}} \right] \right) - \sqrt{a} \sqrt{a-b} \operatorname{ArcTanh} \left[\right. \right. \\
& \left. \left. \frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c + d x]}}{\sqrt{-a-b} \sqrt{-a \operatorname{Cosh}[c + d x]}} \right] \right) \sqrt{-a \operatorname{Cosh}[c + d x]} \sqrt{\frac{-a + a \operatorname{Cosh}[c + d x]}{a + a \operatorname{Cosh}[c + d x]}} (a + a \operatorname{Cosh}[c + d x]) \operatorname{Cosh}[2(c + d x)] \sqrt{\operatorname{Sech}[c + d x]} \right) / \\
& \left. \left(\sqrt{-a-b} \sqrt{a-b} \sqrt{-1 + \operatorname{Cosh}[c + d x]} \sqrt{1 + \operatorname{Cosh}[c + d x]} (a^2 - 2 b^2 + 4 b (b + a \operatorname{Cosh}[c + d x]) - 2 (b + a \operatorname{Cosh}[c + d x])^2) \right) \right) \operatorname{Sech}[c + d x]^{3/2} + \\
& \frac{(b + a \operatorname{Cosh}[c + d x])^2 \left(-\frac{2 b^2}{a^2 (-a^2 + b^2)} - \frac{2 b^3}{a^2 (a^2 - b^2) (b + a \operatorname{Cosh}[c + d x])} \right) \operatorname{Sech}[c + d x]^2}{d (a + b \operatorname{Sech}[c + d x])^{3/2}}
\end{aligned}$$

Problem 146: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + d x]^3}{(a + b \operatorname{Sech}[c + d x])^{3/2}} dx$$

Optimal (type 3, 316 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} - \frac{(2 a-3 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{2(a-b)^{5/2} d} + \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{4(a-b)^{5/2} d} - \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{4(a+b)^{5/2} d} - \\
& \frac{(2 a+3 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{2(a+b)^{5/2} d} - \frac{2 b^4}{a\left(a^2-b^2\right)^2 d \sqrt{a+b \operatorname{Sech}[c+d x]}} - \frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{4(a+b)^2 d(1-\operatorname{Sech}[c+d x])} - \frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{4(a-b)^2 d(1+\operatorname{Sech}[c+d x])}
\end{aligned}$$

Result (type 3, 1019 leaves):

$$\begin{aligned}
& \frac{1}{4 a (a-b)^2 (a+b)^2 d (a+b \operatorname{Sech}[c+d x])^{3/2}} (b+a \operatorname{Cosh}[c+d x])^{3/2} \\
& \left(\left((-a^3 b + 7 a b^3) \left(\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x]}}{\sqrt{-a-b} \sqrt{a \operatorname{Cosh}[c+d x]}} \right] + \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x]}}{\sqrt{a-b} \sqrt{a \operatorname{Cosh}[c+d x]}} \right] \right) \sqrt{\frac{-a+a \operatorname{Cosh}[c+d x]}{a+a \operatorname{Cosh}[c+d x]}} \right. \right. \\
& \left. \left. (a+a \operatorname{Cosh}[c+d x]) \right) \right) / \left(\sqrt{a} \sqrt{-a-b} \sqrt{a-b} \sqrt{-1+\operatorname{Cosh}[c+d x]} \sqrt{a \operatorname{Cosh}[c+d x]} \sqrt{1+\operatorname{Cosh}[c+d x]} \sqrt{\operatorname{Sech}[c+d x]} \right) + \\
& \left((2 a^4 - 6 a^2 b^2 - 2 b^4) \left(\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x]}}{\sqrt{-a-b} \sqrt{a \operatorname{Cosh}[c+d x]}} \right] - \sqrt{-a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x]}}{\sqrt{a-b} \sqrt{a \operatorname{Cosh}[c+d x]}} \right] \right) \sqrt{a \operatorname{Cosh}[c+d x]} \right. \\
& \left. \sqrt{\frac{-a+a \operatorname{Cosh}[c+d x]}{a+a \operatorname{Cosh}[c+d x]}} (a+a \operatorname{Cosh}[c+d x]) \sqrt{\operatorname{Sech}[c+d x]} \right) / \left(a^{3/2} \sqrt{-a-b} \sqrt{a-b} \sqrt{-1+\operatorname{Cosh}[c+d x]} \sqrt{1+\operatorname{Cosh}[c+d x]} \right) + \\
& \left((2 a^4 - 4 a^2 b^2 + 2 b^4) \left(\sqrt{-a-b} \left(-4 \sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{b+a \operatorname{Cosh}[c+d x]}}{\sqrt{-a \operatorname{Cosh}[c+d x]}} \right] + \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x]}}{\sqrt{a-b} \sqrt{-a \operatorname{Cosh}[c+d x]}} \right] \right) - \sqrt{a} \sqrt{a-b} \operatorname{ArcTanh}\left[\right. \right. \\
& \left. \left. \frac{\sqrt{a} \sqrt{b+a \operatorname{Cosh}[c+d x]}}{\sqrt{-a-b} \sqrt{-a \operatorname{Cosh}[c+d x]}} \right] \right) \sqrt{-a \operatorname{Cosh}[c+d x]} \sqrt{\frac{-a+a \operatorname{Cosh}[c+d x]}{a+a \operatorname{Cosh}[c+d x]}} (a+a \operatorname{Cosh}[c+d x]) \operatorname{Cosh}[2(c+d x)] \sqrt{\operatorname{Sech}[c+d x]} \right) / \\
& \left. \left(\sqrt{-a-b} \sqrt{a-b} \sqrt{-1+\operatorname{Cosh}[c+d x]} \sqrt{1+\operatorname{Cosh}[c+d x]} (a^2 - 2 b^2 + 4 b (b+a \operatorname{Cosh}[c+d x]) - 2 (b+a \operatorname{Cosh}[c+d x])^2) \right) \right) \operatorname{Sech}[c+d x]^{3/2} + \\
& \frac{1}{d (a+b \operatorname{Sech}[c+d x])^{3/2}} (b+a \operatorname{Cosh}[c+d x])^2 \left(-\frac{a^4 + a^2 b^2 + 4 b^4}{2 a^2 (-a^2 + b^2)^2} + \frac{2 b^5}{a^2 (a^2 - b^2)^2 (b+a \operatorname{Cosh}[c+d x])} + \right. \\
& \left. \frac{(-a^2 - b^2 + 2 a b \operatorname{Cosh}[c+d x]) \operatorname{Csch}[c+d x]^2}{2 (-a^2 + b^2)^2} \right) \operatorname{Sech}[c+d x]^2
\end{aligned}$$

Problem 147: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Tanh}[c+d x]^4}{(a+b \operatorname{Sech}[c+d x])^{3/2}} dx$$

Optimal (type 4, 907 leaves, 17 steps):

$$\begin{aligned}
& \frac{2 \operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sech}[c+d x])}{a-b}}}{a \sqrt{a+b} d} + \\
& \frac{4 a \operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sech}[c+d x])}{a-b}}}{b^2 \sqrt{a+b} d} - \frac{1}{3 b^4 \sqrt{a+b} d} \\
& \frac{2 a (8 a^2 - 5 b^2) \operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sech}[c+d x])}{a-b}}}{a \sqrt{a+b} d} + \\
& \frac{2 \operatorname{Coth}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sech}[c+d x])}{a-b}}}{b \sqrt{a+b} d} - \frac{1}{3 b^3 \sqrt{a+b} d} \\
& \frac{2 (2 a + b) (4 a + b) \operatorname{Coth}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sech}[c+d x])}{a-b}}}{a^2 d} - \frac{4 a \operatorname{Tanh}[c + d x]}{(a^2 - b^2) d \sqrt{a+b \operatorname{Sech}[c+d x]}} + \\
& \frac{2 b^2 \operatorname{Tanh}[c + d x]}{a (a^2 - b^2) d \sqrt{a+b \operatorname{Sech}[c+d x]}} - \frac{2 a^2 \operatorname{Sech}[c + d x] \operatorname{Tanh}[c + d x]}{b (a^2 - b^2) d \sqrt{a+b \operatorname{Sech}[c+d x]}} + \frac{2 (4 a^2 - b^2) \sqrt{a+b \operatorname{Sech}[c+d x]} \operatorname{Tanh}[c + d x]}{3 b^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 148: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Tanh}[c + d x]^2}{(a + b \operatorname{Sech}[c + d x])^{3/2}} dx$$

Optimal (type 4, 344 leaves, 7 steps):

$$\frac{1}{a b^2 d} 2 (a - b) \sqrt{a + b} \operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}} +$$

$$\frac{2 \sqrt{a + b} \operatorname{Coth}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{a b d} +$$

$$\frac{2 \sqrt{a + b} \operatorname{Coth}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{a^2 d} - \frac{2 \operatorname{Tanh}[c + d x]}{a d \sqrt{a + b \operatorname{Sech}[c + d x]}}$$

Result (type 1, 1 leaves):

???

Problem 149: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(a + b \operatorname{Sech}[c + d x])^{3/2}} dx$$

Optimal (type 4, 347 leaves, 6 steps):

$$-\frac{2 \operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{a \sqrt{a + b} d} +$$

$$\frac{2 \operatorname{Coth}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{a \sqrt{a + b} d} +$$

$$\frac{2 \sqrt{a + b} \operatorname{Coth}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sech}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sech}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sech}[c + d x])}{a - b}}}{a^2 d} + \frac{2 b^2 \operatorname{Tanh}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \operatorname{Sech}[c + d x]}}$$

Result (type 1, 1 leaves):

???

Problem 150: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Coth}[c + d x]^2}{(a + b \operatorname{Sech}[c + d x])^{3/2}} dx$$

Optimal (type 4, 665 leaves, 14 steps):

$$\frac{4 a \operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+d x])}{a-b}}}{(a-b)(a+b)^{3/2} d} -$$

$$\frac{2 \operatorname{Coth}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+d x])}{a-b}}}{a \sqrt{a+b} d} -$$

$$\frac{(3 a - b) \operatorname{Coth}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+d x])}{a-b}}}{(a-b)(a+b)^{3/2} d} +$$

$$\frac{2 \operatorname{Coth}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+d x])}{a-b}}}{a \sqrt{a+b} d} +$$

$$\frac{2 \sqrt{a+b} \operatorname{Coth}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sech}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sech}[c+d x])}{a-b}}}{a^2 d} -$$

$$\frac{\operatorname{Coth}[c + d x]}{d (a + b \operatorname{Sech}[c + d x])^{3/2}} - \frac{b^2 \operatorname{Tanh}[c + d x]}{(a^2 - b^2) d (a + b \operatorname{Sech}[c + d x])^{3/2}} - \frac{4 a b^2 \operatorname{Tanh}[c + d x]}{(a^2 - b^2)^2 d \sqrt{a + b \operatorname{Sech}[c + d x]}} + \frac{2 b^2 \operatorname{Tanh}[c + d x]}{a (a^2 - b^2) d \sqrt{a + b \operatorname{Sech}[c + d x]}}$$

Result (type 1, 1 leaves):

???

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5}{\sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}} dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$\frac{2x^2}{21c^4\sqrt{\text{Sech}[2\text{Log}[cx]]}} + \frac{x^6}{7\sqrt{\text{Sech}[2\text{Log}[cx]]}} + \frac{\sqrt{\frac{c^4 + \frac{1}{x^4}}{(c^2 + \frac{1}{x^2})^2}} \left(c^2 + \frac{1}{x^2}\right) \text{EllipticF}\left[2\text{ArcCot}[cx], \frac{1}{2}\right]}{21c^5\left(c^4 + \frac{1}{x^4}\right)x\sqrt{\text{Sech}[2\text{Log}[cx]]}}$$

Result (type 4, 120 leaves):

$$\frac{1}{21\sqrt{2}c^6\sqrt{c^2x^2}} \sqrt{\frac{c^2x^2}{1+c^4x^4}} \left(\sqrt{c^2x^2} (2+5c^4x^4+3c^8x^8) + 2(-1)^{1/4}\sqrt{1+c^4x^4} \text{EllipticF}\left[i\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right], -1\right] \right)$$

Problem 160: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{\text{Sech}[2\text{Log}[cx]]}} dx$$

Optimal (type 4, 203 leaves, 8 steps):

$$\frac{2}{5c^4\sqrt{\text{Sech}[2\text{Log}[cx]]}} - \frac{2}{5c^4\left(c^2 + \frac{1}{x^2}\right)x^2\sqrt{\text{Sech}[2\text{Log}[cx]]}} + \frac{x^4}{5\sqrt{\text{Sech}[2\text{Log}[cx]]}} +$$

$$\frac{2\sqrt{\frac{c^4 + \frac{1}{x^4}}{(c^2 + \frac{1}{x^2})^2}} \left(c^2 + \frac{1}{x^2}\right) \text{EllipticE}\left[2\text{ArcCot}[cx], \frac{1}{2}\right]}{5c^3\left(c^4 + \frac{1}{x^4}\right)x\sqrt{\text{Sech}[2\text{Log}[cx]]}} - \frac{\sqrt{\frac{c^4 + \frac{1}{x^4}}{(c^2 + \frac{1}{x^2})^2}} \left(c^2 + \frac{1}{x^2}\right) \text{EllipticF}\left[2\text{ArcCot}[cx], \frac{1}{2}\right]}{5c^3\left(c^4 + \frac{1}{x^4}\right)x\sqrt{\text{Sech}[2\text{Log}[cx]]}}$$

Result (type 4, 155 leaves):

$$\frac{1}{5\sqrt{2}c^4\sqrt{c^2x^2}} \sqrt{\frac{c^2x^2}{1+c^4x^4}} \left((c^2x^2)^{3/2} (1+c^4x^4) - \right.$$

$$\left. 2(-1)^{3/4}\sqrt{1+c^4x^4} \text{EllipticE}\left[i\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right], -1\right] + 2(-1)^{3/4}\sqrt{1+c^4x^4} \text{EllipticF}\left[i\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right], -1\right] \right)$$

Problem 162: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{\text{Sech}[2\text{Log}[cx]]}} dx$$

Optimal (type 4, 87 leaves, 5 steps):

$$\frac{x^2}{3 \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}} - \frac{\sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right]}{3 c \left(c^2 + \frac{1}{x^2}\right) x \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}}$$

Result (type 4, 107 leaves):

$$\frac{x^2 \sqrt{\frac{c^2 x^2}{2 + 2 c^4 x^4}} \left(\sqrt{c^2 x^2} (1 + c^4 x^4) - 2 (-1)^{1/4} \sqrt{1 + c^4 x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right], -1\right]\right)}{3 (c^2 x^2)^{3/2}}$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}}{x^3} dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$-\frac{\left(c^4 + \frac{1}{x^4}\right) \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}}{c^2 + \frac{1}{x^2}} + c \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) x \operatorname{EllipticE}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right] \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]} -$$

$$\frac{1}{2} c \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) x \operatorname{EllipticF}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right] \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}$$

Result (type 4, 53 leaves):

$$-c^2 \sqrt{\operatorname{Cosh}[2 \operatorname{Log}[c x]]} \left(\sqrt{\operatorname{Cosh}[2 \operatorname{Log}[c x]]} + i \operatorname{EllipticE}\left[i \operatorname{Log}[c x], 2\right]\right) \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}}{x^5} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{1}{3} \left(c^4 + \frac{1}{x^4}\right) \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]} + \frac{1}{6} c^3 \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) x \operatorname{EllipticF}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right] \sqrt{\operatorname{Sech}[2 \operatorname{Log}[c x]]}$$

Result (type 4, 117 leaves):

$$\frac{1}{3 x^4 \sqrt{c^2 x^2}} \sqrt{2} \sqrt{\frac{c^2 x^2}{1 + c^4 x^4}} \left(-\sqrt{c^2 x^2} (1 + c^4 x^4) + (-1)^{1/4} c^4 x^4 \sqrt{1 + c^4 x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right], -1\right] \right)$$

Problem 170: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 141 leaves, 7 steps):

$$\frac{4}{77 c^4 \left(c^4 + \frac{1}{x^4}\right) \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{6 x^4}{77 \left(c^4 + \frac{1}{x^4}\right) \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^8}{11 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{2 \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right]}{77 c^5 \left(c^4 + \frac{1}{x^4}\right)^2 x^3 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 4, 128 leaves):

$$\frac{1}{154 \sqrt{2} c^8 \sqrt{c^2 x^2}} \sqrt{\frac{c^2 x^2}{1 + c^4 x^4}} \left(\sqrt{c^2 x^2} (4 + 17 c^4 x^4 + 20 c^8 x^8 + 7 c^{12} x^{12}) + 4 (-1)^{1/4} \sqrt{1 + c^4 x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right], -1\right] \right)$$

Problem 172: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5}{\operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$-\frac{4}{15 c^4 \left(c^4 + \frac{1}{x^4}\right) \left(c^2 + \frac{1}{x^2}\right) x^4 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{4}{15 c^4 \left(c^4 + \frac{1}{x^4}\right) x^2 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{2 x^2}{15 \left(c^4 + \frac{1}{x^4}\right) \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^6}{9 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{4 \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) \operatorname{EllipticE}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right]}{15 c^3 \left(c^4 + \frac{1}{x^4}\right)^2 x^3 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{2 \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right]}{15 c^3 \left(c^4 + \frac{1}{x^4}\right)^2 x^3 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 4, 164 leaves):

$$\frac{1}{90 \sqrt{2} c^6 \sqrt{c^2 x^2}} \sqrt{\frac{c^2 x^2}{1 + c^4 x^4}} \left((c^2 x^2)^{3/2} (11 + 16 c^4 x^4 + 5 c^8 x^8) - 12 (-1)^{3/4} \sqrt{1 + c^4 x^4} \operatorname{EllipticE}\left[\frac{i}{2} \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right], -1\right] + 12 (-1)^{3/4} \sqrt{1 + c^4 x^4} \operatorname{EllipticF}\left[\frac{i}{2} \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right], -1\right] \right)$$

Problem 174: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 111 leaves, 6 steps):

$$\frac{2}{7 \left(c^4 + \frac{1}{x^4}\right) \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^4}{7 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{2 \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right]}{7 c \left(c^4 + \frac{1}{x^4}\right)^2 x^3 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 4, 119 leaves):

$$\frac{1}{14 \sqrt{2} c^4 \sqrt{c^2 x^2}} \sqrt{\frac{c^2 x^2}{1 + c^4 x^4}} \left(\sqrt{c^2 x^2} (3 + 4 c^4 x^4 + c^8 x^8) - 4 (-1)^{1/4} \sqrt{1 + c^4 x^4} \operatorname{EllipticF}\left[\frac{i}{2} \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right], -1\right] \right)$$

Problem 176: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 214 leaves, 8 steps):

$$-\frac{12}{5 \left(c^4 + \frac{1}{x^4}\right) \left(c^2 + \frac{1}{x^2}\right) x^4 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{6}{5 \left(c^4 + \frac{1}{x^4}\right) x^2 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^2}{5 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{12 c \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) \operatorname{EllipticE}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right]}{5 \left(c^4 + \frac{1}{x^4}\right)^2 x^3 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{6 c \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2}\right)^2}} \left(c^2 + \frac{1}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}[c x], \frac{1}{2}\right]}{5 \left(c^4 + \frac{1}{x^4}\right)^2 x^3 \operatorname{Sech}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 4, 171 leaves):

$$\frac{1}{10 c^2 (c^2 x^2)^{3/2}} \sqrt{\frac{c^2 x^2}{2 + 2 c^4 x^4}} \left(\sqrt{c^2 x^2} (-5 - 4 c^4 x^4 + c^8 x^8) - 12 (-1)^{3/4} c^2 x^2 \sqrt{1 + c^4 x^4} \text{EllipticE} \left[i \text{ArcSinh} \left[(-1)^{1/4} \sqrt{c^2 x^2} \right], -1 \right] + 12 (-1)^{3/4} c^2 x^2 \sqrt{1 + c^4 x^4} \text{EllipticF} \left[i \text{ArcSinh} \left[(-1)^{1/4} \sqrt{c^2 x^2} \right], -1 \right] \right)$$

Problem 180: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sech} [2 \text{Log} [c x]]^{3/2}}{x^3} dx$$

Optimal (type 4, 92 leaves, 5 steps):

$$\frac{1}{2} \left(c^4 + \frac{1}{x^4} \right) x^2 \text{Sech} [2 \text{Log} [c x]]^{3/2} - \frac{\left(c^4 + \frac{1}{x^4} \right) \sqrt{\frac{c^4 + \frac{1}{x^4}}{\left(c^2 + \frac{1}{x^2} \right)^2}} \left(c^2 + \frac{1}{x^2} \right) x^3 \text{EllipticF} \left[2 \text{ArcCot} [c x], \frac{1}{2} \right] \text{Sech} [2 \text{Log} [c x]]^{3/2}}{4 c}$$

Result (type 4, 98 leaves):

$$\frac{\sqrt{2} c^2 \sqrt{\frac{c^2 x^2}{1 + c^4 x^4}} \left(\sqrt{c^2 x^2} - (-1)^{1/4} \sqrt{1 + c^4 x^4} \text{EllipticF} \left[i \text{ArcSinh} \left[(-1)^{1/4} \sqrt{c^2 x^2} \right], -1 \right] \right)}{\sqrt{c^2 x^2}}$$

Problem 185: Result more than twice size of optimal antiderivative.

$$\int \text{Sech} [a + b \text{Log} [c x^n]]^4 dx$$

Optimal (type 5, 69 leaves, 4 steps):

$$\frac{16 e^{4a} x (c x^n)^{4b} \text{Hypergeometric2F1} \left[4, \frac{1}{2} \left(4 + \frac{1}{bn} \right), \frac{1}{2} \left(6 + \frac{1}{bn} \right), -e^{2a} (c x^n)^{2b} \right]}{1 + 4 b n}$$

Result (type 5, 750 leaves):

$$\begin{aligned}
& \frac{1}{6 b^3 n^3} (-1 + 4 b^2 n^2) x \operatorname{Sech}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sech}[a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sinh}[b n \operatorname{Log}[x]] + \\
& \frac{1}{3 b n} x \operatorname{Sech}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sech}[a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^3 \operatorname{Sinh}[b n \operatorname{Log}[x]] + \\
& \frac{1}{6 b^2 n^2} x \operatorname{Sech}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sech}[a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^2 \\
& (\operatorname{Cosh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 2 b n \operatorname{Sinh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]) + \\
& \frac{1}{6 b^3 n^3 (1 + 2 b n)} e^{-\frac{a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n])}{b n}} \operatorname{Sech}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \left(e^{\left(2+\frac{1}{b n}\right)(a+b \operatorname{Log}[c x^n])} \operatorname{Cosh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{2 b n}, 2 + \frac{1}{2 b n}, -e^{2(a+b \operatorname{Log}[c x^n])}\right] - \right. \\
& \left. e^{\frac{a}{b n} + \frac{-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]}{n}} (1 + 2 b n) x \left(\operatorname{Cosh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2 b n}, 1 + \frac{1}{2 b n}, -e^{2(a+b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}\right] + \right. \right. \\
& \left. \left. \operatorname{Sinh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]\right) \right) - \frac{1}{3 b n (1 + 2 b n)} 2 e^{-\frac{a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n])}{b n}} \operatorname{Sech}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \left(e^{\left(2+\frac{1}{b n}\right)(a+b \operatorname{Log}[c x^n])} \operatorname{Cosh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{2 b n}, 2 + \frac{1}{2 b n}, -e^{2(a+b \operatorname{Log}[c x^n])}\right] - \right. \\
& \left. e^{\frac{a}{b n} + \frac{-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]}{n}} (1 + 2 b n) x \left(\operatorname{Cosh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right. \right. \\
& \left. \left. \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2 b n}, 1 + \frac{1}{2 b n}, -e^{2(a+b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}\right] + \operatorname{Sinh}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]\right) \right) \right)
\end{aligned}$$

Problem 187: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[a + 2 \operatorname{Log}[c \sqrt{x}]]^3 dx$$

Optimal (type 1, 25 leaves, 3 steps):

$$\frac{2 c^6 e^{-a}}{\left(c^4 + \frac{e^{-2a}}{x^2}\right)^2}$$

Result (type 1, 62 leaves):

$$\frac{2 (\operatorname{Cosh}[a] - \operatorname{Sinh}[a]) (2 c^4 x^2 + \operatorname{Cosh}[a]^2 - 2 \operatorname{Cosh}[a] \operatorname{Sinh}[a] + \operatorname{Sinh}[a]^2)}{c^2 ((1 + c^4 x^2) \operatorname{Cosh}[a] + (-1 + c^4 x^2) \operatorname{Sinh}[a])^2}$$

Problem 188: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}\left[a + 2 \operatorname{Log}\left[\frac{c}{\sqrt{x}}\right]\right]^3 dx$$

Optimal (type 1, 25 leaves, 4 steps):

$$\frac{2 c^2 e^{-3 a}}{\left(e^{-2 a} + \frac{c^4}{x^2}\right)^2}$$

Result (type 1, 64 leaves):

$$-\frac{2 c^6 \left((c^4 + 2 x^2) \operatorname{Cosh}[a] + (c^4 - 2 x^2) \operatorname{Sinh}[a] \right) \left(\operatorname{Cosh}[2 a] + \operatorname{Sinh}[2 a] \right)}{\left((c^4 + x^2) \operatorname{Cosh}[a] + (c^4 - x^2) \operatorname{Sinh}[a] \right)^2}$$

Test results for the 220 problems in "6.5.7 (d hyper)^m (a+b (c sech)^n)^p.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + d x] \left(a + b \operatorname{Sech}[c + d x]^2 \right) dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$-\frac{(a + b) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{d} + \frac{b \operatorname{Sech}[c + d x]}{d}$$

Result (type 3, 84 leaves):

$$-\frac{a \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} - \frac{b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{b \operatorname{Sech}[c + d x]}{d}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + d x]^3 \left(a + b \operatorname{Sech}[c + d x]^2 \right) dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{(a + 3 b) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{2 d} - \frac{(a + b) \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]}{2 d} - \frac{b \operatorname{Sech}[c + d x]}{d}$$

Result (type 3, 169 leaves):

$$\begin{aligned}
& - \frac{a \operatorname{Csch}\left[\frac{1}{2}(c+dx)\right]^2}{8d} - \frac{b \operatorname{Csch}\left[\frac{1}{2}(c+dx)\right]^2}{8d} + \frac{a \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{3b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} \\
& - \frac{a \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{3b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{a \operatorname{Sech}\left[\frac{1}{2}(c+dx)\right]^2}{8d} - \frac{b \operatorname{Sech}\left[\frac{1}{2}(c+dx)\right]^2}{8d} - \frac{b \operatorname{Sech}[c+dx]}{d}
\end{aligned}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx] (a+b \operatorname{Sech}[c+dx]^2)^2 dx$$

Optimal (type 3, 52 leaves, 4 steps):

$$- \frac{(a+b)^2 \operatorname{ArcTanh}\left[\operatorname{Cosh}[c+dx]\right]}{d} + \frac{b(2a+b) \operatorname{Sech}[c+dx]}{d} + \frac{b^2 \operatorname{Sech}[c+dx]^3}{3d}$$

Result (type 3, 108 leaves):

$$\begin{aligned}
& - \left(\left(4(b+a \operatorname{Cosh}[c+dx]^2)^2 \left(-b^2 - 3b(2a+b) \operatorname{Cosh}[c+dx]^2 + 3(a+b)^2 \operatorname{Cosh}[c+dx]^3 \left(\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] \right) \right) \right) \right) \\
& \operatorname{Sech}[c+dx]^3 \Big/ \left(3d(a+2b+a \operatorname{Cosh}[2(c+dx)])^2 \right)
\end{aligned}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx]^2 (a+b \operatorname{Sech}[c+dx]^2)^2 dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$- \frac{(a+b)^2 \operatorname{Coth}[c+dx]}{d} - \frac{2b(a+b) \operatorname{Tanh}[c+dx]}{d} + \frac{b^2 \operatorname{Tanh}[c+dx]^3}{3d}$$

Result (type 3, 109 leaves):

$$\begin{aligned}
& - \left(\left(4(b+a \operatorname{Cosh}[c+dx]^2)^2 \operatorname{Sech}[c+dx]^3 \right. \right. \\
& \left. \left(b^2 \operatorname{Sech}[c] \operatorname{Sinh}[dx] + \operatorname{Cosh}[c+dx]^2 \left(-3(a+b)^2 \operatorname{Coth}[c+dx] \operatorname{CsCh}[c] + b(6a+5b) \operatorname{Sech}[c] \right) \operatorname{Sinh}[dx] + b^2 \operatorname{Cosh}[c+dx] \operatorname{Tanh}[c] \right) \right) \Big/ \left(3d(a+2b+a \operatorname{Cosh}[2(c+dx)])^2 \right)
\end{aligned}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx]^4 (a+b \operatorname{Sech}[c+dx]^2)^2 dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$\frac{(a+b)(a+3b)\operatorname{Coth}[c+dx]}{d} - \frac{(a+b)^2\operatorname{Coth}[c+dx]^3}{3d} + \frac{b(2a+3b)\operatorname{Tanh}[c+dx]}{d} - \frac{b^2\operatorname{Tanh}[c+dx]^3}{3d}$$

Result (type 3, 151 leaves):

$$-\frac{1}{6d} \operatorname{Csch}[2c] \operatorname{Csch}[2(c+dx)]^3 \left(8a(a+2b)\operatorname{Sinh}[2c] - 6(a+2b)^2\operatorname{Sinh}[2dx] - 3a^2\operatorname{Sinh}[2(c+dx)] - 6ab\operatorname{Sinh}[2(c+dx)] + a^2\operatorname{Sinh}[6(c+dx)] + 2ab\operatorname{Sinh}[6(c+dx)] + 3a^2\operatorname{Sinh}[4c+2dx] + a^2\operatorname{Sinh}[4c+6dx] + 8ab\operatorname{Sinh}[4c+6dx] + 8b^2\operatorname{Sinh}[4c+6dx] \right)$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int (a+b\operatorname{Sech}[c+dx]^2)^3 \operatorname{Sinh}[c+dx]^4 dx$$

Optimal (type 3, 182 leaves, 6 steps):

$$\frac{3}{8} a (a^2 - 12ab + 8b^2) x - \frac{3a(a^2 - 12ab + 8b^2)\operatorname{Tanh}[c+dx]}{8d} + \frac{b(6a^2 - 23ab - 8b^2)\operatorname{Tanh}[c+dx]^3}{8d} - \frac{3(5a - 16b)b^2\operatorname{Tanh}[c+dx]^5}{40d} - \frac{3(a-2b)\operatorname{Sinh}[c+dx]^2\operatorname{Tanh}[c+dx](a+b-b\operatorname{Tanh}[c+dx]^2)^2}{8d} + \frac{\operatorname{Cosh}[c+dx]\operatorname{Sinh}[c+dx]^3(a+b-b\operatorname{Tanh}[c+dx]^2)^3}{4d}$$

Result (type 3, 651 leaves):

$$\frac{1}{1280d(a+2b+a\operatorname{Cosh}[2(c+dx)])^3} \left((b+a\operatorname{Cosh}[c+dx]^2)^3 \operatorname{Sech}[c] \operatorname{Sech}[c+dx]^5 (1200a(a^2-12ab+8b^2)dx\operatorname{Cosh}[dx] + 1200a(a^2-12ab+8b^2)dx\operatorname{Cosh}[2c+dx] + 600a^3dx\operatorname{Cosh}[2c+3dx] - 7200a^2b dx\operatorname{Cosh}[2c+3dx] + 4800a^2b^2 dx\operatorname{Cosh}[2c+3dx] + 600a^3dx\operatorname{Cosh}[4c+3dx] - 7200a^2b dx\operatorname{Cosh}[4c+3dx] + 4800a^2b^2 dx\operatorname{Cosh}[4c+3dx] + 120a^3dx\operatorname{Cosh}[4c+5dx] - 1440a^2b dx\operatorname{Cosh}[4c+5dx] + 960a^2b^2 dx\operatorname{Cosh}[4c+5dx] + 120a^3dx\operatorname{Cosh}[6c+5dx] - 1440a^2b dx\operatorname{Cosh}[6c+5dx] + 960a^2b^2 dx\operatorname{Cosh}[6c+5dx] - 180a^3\operatorname{Sinh}[dx] + 12120a^2b\operatorname{Sinh}[dx] - 14080a^2b^2\operatorname{Sinh}[dx] + 1280b^3\operatorname{Sinh}[dx] - 180a^3\operatorname{Sinh}[2c+dx] - 7080a^2b\operatorname{Sinh}[2c+dx] + 11520a^2b^2\operatorname{Sinh}[2c+dx] - 310a^3\operatorname{Sinh}[2c+3dx] + 8760a^2b\operatorname{Sinh}[2c+3dx] - 8960a^2b^2\operatorname{Sinh}[2c+3dx] - 310a^3\operatorname{Sinh}[4c+3dx] - 840a^2b\operatorname{Sinh}[4c+3dx] + 3840a^2b^2\operatorname{Sinh}[4c+3dx] - 640b^3\operatorname{Sinh}[4c+3dx] - 150a^3\operatorname{Sinh}[4c+5dx] + 2520a^2b\operatorname{Sinh}[4c+5dx] - 2560a^2b^2\operatorname{Sinh}[4c+5dx] + 128b^3\operatorname{Sinh}[4c+5dx] - 150a^3\operatorname{Sinh}[6c+5dx] + 600a^2b\operatorname{Sinh}[6c+5dx] - 15a^3\operatorname{Sinh}[6c+7dx] + 120a^2b\operatorname{Sinh}[6c+7dx] - 15a^3\operatorname{Sinh}[8c+7dx] + 120a^2b\operatorname{Sinh}[8c+7dx] + 5a^3\operatorname{Sinh}[8c+9dx] + 5a^3\operatorname{Sinh}[10c+9dx]) \right)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int (a+b\operatorname{Sech}[c+dx]^2)^3 \operatorname{Sinh}[c+dx]^2 dx$$

Optimal (type 3, 112 leaves, 6 steps):

$$-\frac{1}{2} a^2 (a - 6b) x + \frac{a^3}{4d(1 - \operatorname{Tanh}[c + dx])} - \frac{3a^2 b \operatorname{Tanh}[c + dx]}{d} + \frac{b^2(3a + b) \operatorname{Tanh}[c + dx]^3}{3d} - \frac{b^3 \operatorname{Tanh}[c + dx]^5}{5d} - \frac{a^3}{4d(1 + \operatorname{Tanh}[c + dx])}$$

Result (type 3, 480 leaves):

$$\frac{1}{3840d} \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^5$$

$$\begin{aligned} & (-600a^2(a - 6b)dx \operatorname{Cosh}[dx] - 600a^2(a - 6b)dx \operatorname{Cosh}[2c + dx] - 300a^3dx \operatorname{Cosh}[2c + 3dx] + 1800a^2b dx \operatorname{Cosh}[2c + 3dx] - \\ & 300a^3dx \operatorname{Cosh}[4c + 3dx] + 1800a^2b dx \operatorname{Cosh}[4c + 3dx] - 60a^3dx \operatorname{Cosh}[4c + 5dx] + 360a^2b dx \operatorname{Cosh}[4c + 5dx] - \\ & 60a^3dx \operatorname{Cosh}[6c + 5dx] + 360a^2b dx \operatorname{Cosh}[6c + 5dx] + 75a^3 \operatorname{Sinh}[dx] - 4320a^2b \operatorname{Sinh}[dx] + 960a^2b^2 \operatorname{Sinh}[dx] - \\ & 160b^3 \operatorname{Sinh}[dx] + 75a^3 \operatorname{Sinh}[2c + dx] + 2880a^2b \operatorname{Sinh}[2c + dx] - 1440a^2b^2 \operatorname{Sinh}[2c + dx] - 480b^3 \operatorname{Sinh}[2c + dx] + \\ & 135a^3 \operatorname{Sinh}[2c + 3dx] - 2880a^2b \operatorname{Sinh}[2c + 3dx] + 480a^2b^2 \operatorname{Sinh}[2c + 3dx] + 160b^3 \operatorname{Sinh}[2c + 3dx] + \\ & 135a^3 \operatorname{Sinh}[4c + 3dx] + 720a^2b \operatorname{Sinh}[4c + 3dx] - 720a^2b^2 \operatorname{Sinh}[4c + 3dx] + 75a^3 \operatorname{Sinh}[4c + 5dx] - 720a^2b \operatorname{Sinh}[4c + 5dx] + \\ & 240a^2b^2 \operatorname{Sinh}[4c + 5dx] + 32b^3 \operatorname{Sinh}[4c + 5dx] + 75a^3 \operatorname{Sinh}[6c + 5dx] + 15a^3 \operatorname{Sinh}[6c + 7dx] + 15a^3 \operatorname{Sinh}[8c + 7dx]) \end{aligned}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + dx]^2 (a + b \operatorname{Sech}[c + dx]^2)^3 dx$$

Optimal (type 3, 70 leaves, 3 steps):

$$-\frac{(a + b)^3 \operatorname{Coth}[c + dx]}{d} - \frac{3b(a + b)^2 \operatorname{Tanh}[c + dx]}{d} + \frac{b^2(a + b) \operatorname{Tanh}[c + dx]^3}{d} - \frac{b^3 \operatorname{Tanh}[c + dx]^5}{5d}$$

Result (type 3, 380 leaves):

$$-\frac{1}{40d(a + 2b + a \operatorname{Cosh}[2(c + dx)])^3}$$

$$\begin{aligned} & \operatorname{Coth}[c + dx] \operatorname{Csch}[c] \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + dx]^2)^3 (10a(5a^2 + 12ab + 8b^2) \operatorname{Sinh}[2c] - 10(5a^3 + 18a^2b + 20ab^2 + 8b^3) \operatorname{Sinh}[2dx] - \\ & 25a^3 \operatorname{Sinh}[2(c + dx)] + 50a^2b \operatorname{Sinh}[2(c + dx)] + 30b^3 \operatorname{Sinh}[2(c + dx)] - 20a^3 \operatorname{Sinh}[4(c + dx)] + 40a^2b \operatorname{Sinh}[4(c + dx)] + \\ & 24b^3 \operatorname{Sinh}[4(c + dx)] - 5a^3 \operatorname{Sinh}[6(c + dx)] + 10a^2b \operatorname{Sinh}[6(c + dx)] + 6b^3 \operatorname{Sinh}[6(c + dx)] - 25a^3 \operatorname{Sinh}[2(c + 2dx)] - \\ & 120a^2b \operatorname{Sinh}[2(c + 2dx)] - 160a^2b^2 \operatorname{Sinh}[2(c + 2dx)] - 64b^3 \operatorname{Sinh}[2(c + 2dx)] + 25a^3 \operatorname{Sinh}[4c + 2dx] + 30a^2b \operatorname{Sinh}[4c + 2dx] + \\ & 5a^3 \operatorname{Sinh}[6c + 4dx] - 5a^3 \operatorname{Sinh}[4c + 6dx] - 30a^2b \operatorname{Sinh}[4c + 6dx] - 40a^2b^2 \operatorname{Sinh}[4c + 6dx] - 16b^3 \operatorname{Sinh}[4c + 6dx]) \end{aligned}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + dx]^3 (a + b \operatorname{Sech}[c + dx]^2)^3 dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{(a+b)^2 (a+7b) \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{2d} - \frac{(a+b)^2 (a+7b) \operatorname{Sech}[c+dx]}{2d} -$$

$$\frac{b(6a^2+15ab+7b^2) \operatorname{Sech}[c+dx]^3}{6d} - \frac{b^2(5a+7b) \operatorname{Sech}[c+dx]^5}{10d} - \frac{(a+b)(b+a \operatorname{Cosh}[c+dx]^2)^2 \operatorname{Csch}[c+dx]^2 \operatorname{Sech}[c+dx]^5}{2d}$$

Result (type 3, 409 leaves):

$$-\frac{1}{120d(a+2b+a \operatorname{Cosh}[2c+2dx])^3}$$

$$(150a^3+270a^2b-30a^2b^2-206b^3+225a^3 \operatorname{Cosh}[2c+2dx]+585a^2b \operatorname{Cosh}[2c+2dx]+495ab^2 \operatorname{Cosh}[2c+2dx]+231b^3 \operatorname{Cosh}[2c+2dx]+$$

$$90a^3 \operatorname{Cosh}[4c+4dx]+450a^2b \operatorname{Cosh}[4c+4dx]+750ab^2 \operatorname{Cosh}[4c+4dx]+350b^3 \operatorname{Cosh}[4c+4dx]+15a^3 \operatorname{Cosh}[6c+6dx]+$$

$$135a^2b \operatorname{Cosh}[6c+6dx]+225ab^2 \operatorname{Cosh}[6c+6dx]+105b^3 \operatorname{Cosh}[6c+6dx]) \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx] (a+b \operatorname{Sech}[c+dx]^2)^3 +$$

$$\frac{4(a^3+9a^2b+15ab^2+7b^3) \operatorname{Cosh}[c+dx]^6 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2}+\frac{dx}{2}\right]\right] (a+b \operatorname{Sech}[c+dx]^2)^3}{d(a+2b+a \operatorname{Cosh}[2c+2dx])^3} -$$

$$\frac{4(a^3+9a^2b+15ab^2+7b^3) \operatorname{Cosh}[c+dx]^6 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2}+\frac{dx}{2}\right]\right] (a+b \operatorname{Sech}[c+dx]^2)^3}{d(a+2b+a \operatorname{Cosh}[2c+2dx])^3}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx]^4 (a+b \operatorname{Sech}[c+dx]^2)^3 dx$$

Optimal (type 3, 104 leaves, 3 steps):

$$\frac{(a+b)^2 (a+4b) \operatorname{Coth}[c+dx]}{d} - \frac{(a+b)^3 \operatorname{Coth}[c+dx]^3}{3d} + \frac{3b(a+b)(a+2b) \operatorname{Tanh}[c+dx]}{d} - \frac{b^2(3a+4b) \operatorname{Tanh}[c+dx]^3}{3d} + \frac{b^3 \operatorname{Tanh}[c+dx]^5}{5d}$$

Result (type 3, 213 leaves):

$$-\frac{1}{15d(a+2b+a \operatorname{Cosh}[2(c+dx)])^3}$$

$$8(b+a \operatorname{Cosh}[c+dx]^2)^3 \operatorname{Sech}[c+dx]^5 (-3b^3 \operatorname{Cosh}[c+dx] + \operatorname{Cosh}[c+dx]^3 (-b^2(15a+14b) + 5(a+b)^3 \operatorname{Coth}[c]^2 \operatorname{Coth}[c+dx]^2) -$$

$$3b^3 \operatorname{Csch}[c] \operatorname{Sinh}[dx] + \operatorname{Cosh}[c+dx]^4 (-b(45a^2+120ab+73b^2) + 5(a+b)^2(2a+11b) \operatorname{Coth}[c] \operatorname{Coth}[c+dx])) \operatorname{Csch}[c] \operatorname{Sinh}[dx] -$$

$$\operatorname{Cosh}[c+dx]^2 (b^2(15a+14b) + 5(a+b)^3 \operatorname{Coth}[c] \operatorname{Coth}[c+dx]^3) \operatorname{Csch}[c] \operatorname{Sinh}[dx]) \operatorname{Tanh}[c]$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c+dx]^4}{a+b \operatorname{Sech}[c+dx]^2} dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{(3 a^2 + 12 a b + 8 b^2) x}{8 a^3} - \frac{\sqrt{b} (a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{a^3 d} - \frac{(5 a + 4 b) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{8 a^2 d} + \frac{\operatorname{Cosh}[c + d x]^3 \operatorname{Sinh}[c + d x]}{4 a d}$$

Result (type 3, 294 leaves):

$$\begin{aligned} & - \frac{1}{64 a^3 \sqrt{b} \sqrt{a + b} d (a + b \operatorname{Sech}[c + d x]^2) \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} (a + 2 b + a \operatorname{Cosh}[2(c + d x)]) \operatorname{Sech}[c + d x]^2 \\ & \left(\sqrt{b} (3 a^3 + 34 a^2 b + 64 a b^2 + 32 b^3) \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])}{2 \sqrt{a + b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}\right] \right. \\ & \quad \left. (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) - \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \left(a^2 (3 a + 2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right] + \right. \right. \\ & \quad \left. \left. \sqrt{b} \sqrt{a + b} (-2 a^2 c + 12 a^2 d x + 48 a b d x + 32 b^2 d x - 8 a (a + b) \operatorname{Sinh}[2(c + d x)] + a^2 \operatorname{Sinh}[4(c + d x)]) \right) \right) \end{aligned}$$

Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + d x]^3}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$\frac{\sqrt{b} (a + b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c + d x]}{\sqrt{b}}\right]}{a^{5/2} d} - \frac{(a + b) \operatorname{Cosh}[c + d x]}{a^2 d} + \frac{\operatorname{Cosh}[c + d x]^3}{3 a d}$$

Result (type 3, 372 leaves):

$$\frac{1}{48 a^{5/2} \sqrt{b} d (b + a \operatorname{Cosh}[c + d x]^2)} \left((a + 2 b + a \operatorname{Cosh}[2(c + d x)]) \left(3 (a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) + 3 (a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) - 3 a^2 \left(\operatorname{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] \right) - 6 \sqrt{a} \sqrt{b} (3 a + 4 b) \operatorname{Cosh}[c + d x] + 2 a^{3/2} \sqrt{b} \operatorname{Cosh}[3(c + d x)] \right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + d x]^2}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 75 leaves, 5 steps):

$$-\frac{(a + 2 b) x}{2 a^2} + \frac{\sqrt{b} \sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{a^2 d} + \frac{\operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d}$$

Result (type 3, 236 leaves):

$$\frac{1}{16 (a + b \operatorname{Sech}[c + d x]^2)} (a + 2 b + a \operatorname{Cosh}[2(c + d x)]) \operatorname{Sech}[c + d x]^2 \left(-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{\sqrt{b} \sqrt{a+b} d} + \frac{1}{a^2} \left(-4 (a + 2 b) x + \left((a^2 + 8 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])}{2 \sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}\right]} (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) \right) / \left(\sqrt{a+b} d \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} + \frac{2 a \operatorname{Cosh}[2 d x] \operatorname{Sinh}[2 c]}{d} + \frac{2 a \operatorname{Cosh}[2 c] \operatorname{Sinh}[2 d x]}{d} \right) \right) \right)$$

Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 47 leaves, 3 steps):

$$-\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c + d x]}{\sqrt{b}}\right]}{a^{3/2} d} + \frac{\text{Cosh}[c + d x]}{a d}$$

Result (type 3, 328 leaves):

$$\frac{1}{8 a^{3/2} d (a + b \text{Sech}[c + d x]^2)} \left(-\frac{1}{\sqrt{b}} (a + 4 b) \left(\text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] (\sqrt{a} - i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right]) \right)\right] + \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] (\sqrt{a} + i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right]) \right)\right] \right) + \frac{a \left(\text{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a + b} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] + \text{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a + b} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] \right)}{\sqrt{b}} + 4 \sqrt{a} \text{Cosh}[c + d x] \right) (a + 2 b + a \text{Cosh}[2(c + d x)]) \text{Sech}[c + d x]^2$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c + d x]}{\sqrt{b}}\right]}{\sqrt{a} (a + b) d} - \frac{\text{ArcTanh}[\text{Cosh}[c + d x]]}{(a + b) d}$$

Result (type 3, 232 leaves):

$$\frac{1}{(a+b)d} \left(\frac{\sqrt{b} \operatorname{ArcTan} \left[\frac{(\sqrt{a}-i\sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Sinh}[c] \operatorname{Tanh} \left[\frac{dx}{2} \right] + \operatorname{Cosh}[c] (\sqrt{a}-i\sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh} \left[\frac{dx}{2} \right]}}{\sqrt{b}} \right]}{\sqrt{a}} \right) + \frac{\sqrt{b} \operatorname{ArcTan} \left[\frac{(\sqrt{a}+i\sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Sinh}[c] \operatorname{Tanh} \left[\frac{dx}{2} \right] + \operatorname{Cosh}[c] (\sqrt{a}+i\sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh} \left[\frac{dx}{2} \right]}}{\sqrt{b}} \right]}{\sqrt{a}} \right) - \left. \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \right] + \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \right] \right)$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]^2}{a+b \operatorname{Sech}[c+dx]^2} dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh} [c+dx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2} d} - \frac{\operatorname{Coth} [c+dx]}{(a+b) d}$$

Result (type 3, 179 leaves):

$$\left((a+2b+a \operatorname{Cosh} [2(c+dx)]) \operatorname{Sech} [c+dx]^2 \right. \\ \left. \left(b \operatorname{ArcTanh} \left[\frac{\operatorname{Sech} [dx] (\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]) ((a+2b) \operatorname{Sinh} [dx] - a \operatorname{Sinh} [2c+dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh} [c] - \operatorname{Sinh} [c])^4} \right] (\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]) + \right. \right. \\ \left. \left. \sqrt{a+b} \operatorname{Csch} [c] \operatorname{Csch} [c+dx] \sqrt{b} (\operatorname{Cosh} [c] - \operatorname{Sinh} [c])^4 \operatorname{Sinh} [dx] \right) \right) \Bigg/ \left(2(a+b)^{3/2} d (a+b \operatorname{Sech} [c+dx]^2) \sqrt{b} (\operatorname{Cosh} [c] - \operatorname{Sinh} [c])^4 \right)$$

Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^3}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$-\frac{\sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c + d x]}{\sqrt{b}}\right]}{(a + b)^2 d} + \frac{(a - b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 (a + b)^2 d} - \frac{\text{Coth}[c + d x] \text{Csch}[c + d x]}{2 (a + b) d}$$

Result (type 3, 338 leaves):

$$\begin{aligned} & -\frac{1}{16 (a + b)^2 d (a + b \text{Sech}[c + d x]^2)} \\ & (a + 2 b + a \text{Cosh}[2 (c + d x)]) \left(8 \sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \right. \right. \right. \\ & \quad \left. \left. \left. \text{Cosh}[c] \left(\sqrt{a} - i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) + 8 \sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \right. \\ & \quad \left. \left. \left((\sqrt{a} + i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} + i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \right) + \\ & (a + b) \text{Csch}\left[\frac{1}{2} (c + d x)\right]^2 - 4 a \text{Log}\left[\text{Cosh}\left[\frac{1}{2} (c + d x)\right]\right] + 4 b \text{Log}\left[\text{Cosh}\left[\frac{1}{2} (c + d x)\right]\right] + 4 a \text{Log}\left[\text{Sinh}\left[\frac{1}{2} (c + d x)\right]\right] - \\ & 4 b \text{Log}\left[\text{Sinh}\left[\frac{1}{2} (c + d x)\right]\right] + (a + b) \text{Sech}\left[\frac{1}{2} (c + d x)\right]^2 \Big) \text{Sech}[c + d x]^2 \end{aligned}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^4}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$-\frac{a \sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{(a + b)^{5/2} d} + \frac{a \text{Coth}[c + d x]}{(a + b)^2 d} - \frac{\text{Coth}[c + d x]^3}{3 (a + b) d}$$

Result (type 3, 216 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^2 \right. \\ \left. \left(3ab \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] (-\operatorname{Cosh}[2c] + \operatorname{Sinh}[2c]) + \right. \right. \\ \left. \left. \frac{1}{4} \sqrt{a+b} \operatorname{Csch}[c] \operatorname{Csch}[c + dx]^3 \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 (6a \operatorname{Sinh}[dx] - 3b \operatorname{Sinh}[2c + dx] + (-2a + b) \operatorname{Sinh}[2c + 3dx]) \right) \right) / \\ \left(6(a+b)^{5/2} d (a + b \operatorname{Sech}[c + dx]^2) \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right)$$

Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + dx]^4}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$\frac{3(a^2 + 8ab + 8b^2)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{2a^4d} - \\ \frac{(5a+6b)\operatorname{Cosh}[c+dx]\operatorname{Sinh}[c+dx]}{8a^2d(a+b-b\operatorname{Tanh}[c+dx]^2)} + \frac{\operatorname{Cosh}[c+dx]^3\operatorname{Sinh}[c+dx]}{4ad(a+b-b\operatorname{Tanh}[c+dx]^2)} - \frac{3b(3a+4b)\operatorname{Tanh}[c+dx]}{8a^3d(a+b-b\operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 1330 leaves):

$$- \left(\left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \right. \right. \\ \left. \left(16x + \left((a^3 - 6a^2b - 24ab^2 - 16b^3) \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] \right. \right. \right. \\ \left. \left. \left. (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \right) / \left(b(a+b)^{3/2} d \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right) + \right. \\ \left. \left. \frac{(a^2 + 8ab + 8b^2) \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{b(a+b)d(a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) \right) / \left(256a^2 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) +$$

$$\begin{aligned}
& \frac{3 (a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \left(\frac{(a+2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8b^{3/2}(a+b)^{3/2}d} - \frac{a \operatorname{Sinh}[2(c+dx)]}{8b(a+b)d(a+2b+a \operatorname{Cosh}[2(c+dx)])} \right)}{128 (a + b \operatorname{Sech}[c + dx]^2)^2} + \\
& \frac{1}{128 (a + b \operatorname{Sech}[c + dx]^2)^2} \\
& (a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \\
& \left(\frac{1}{a+b} (a^5 - 30a^4b - 480a^3b^2 - 1600a^2b^3 - 1920ab^4 - 768b^5) \right. \\
& \left. \left(- \left(\left(\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \right. \\
& \left. \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) / \left(8a^4b\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \right) + \\
& \left(\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
& \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \right) / \left(8a^4b\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \right) + \\
& \frac{1}{8a^4b(a+b)d(a+2b+a \operatorname{Cosh}[2c+2dx])} \operatorname{Sech}[2c] \left(160a^4bdx \operatorname{Cosh}[2c] + 1248a^3b^2dx \operatorname{Cosh}[2c] + 3392a^2b^3dx \operatorname{Cosh}[2c] + \right. \\
& 3840a^4bdx \operatorname{Cosh}[2c] + 1536b^5dx \operatorname{Cosh}[2c] + 80a^4bdx \operatorname{Cosh}[2dx] + 464a^3b^2dx \operatorname{Cosh}[2dx] + 768a^2b^3dx \operatorname{Cosh}[2dx] + \\
& 384a^4bdx \operatorname{Cosh}[2dx] + 80a^4bdx \operatorname{Cosh}[4c+2dx] + 464a^3b^2dx \operatorname{Cosh}[4c+2dx] + 768a^2b^3dx \operatorname{Cosh}[4c+2dx] + \\
& 384a^4bdx \operatorname{Cosh}[4c+2dx] + a^5 \operatorname{Sinh}[2c] + 34a^4b \operatorname{Sinh}[2c] + 224a^3b^2 \operatorname{Sinh}[2c] + 576a^2b^3 \operatorname{Sinh}[2c] + 640a^4b^4 \operatorname{Sinh}[2c] + \\
& 256b^5 \operatorname{Sinh}[2c] - a^5 \operatorname{Sinh}[2dx] - 62a^4b \operatorname{Sinh}[2dx] - 318a^3b^2 \operatorname{Sinh}[2dx] - 512a^2b^3 \operatorname{Sinh}[2dx] - 256a^4b^4 \operatorname{Sinh}[2dx] - \\
& 30a^4b \operatorname{Sinh}[4c+2dx] - 158a^3b^2 \operatorname{Sinh}[4c+2dx] - 256a^2b^3 \operatorname{Sinh}[4c+2dx] - 128a^4b^4 \operatorname{Sinh}[4c+2dx] - \\
& 12a^4b \operatorname{Sinh}[2c+4dx] - 36a^3b^2 \operatorname{Sinh}[2c+4dx] - 24a^2b^3 \operatorname{Sinh}[2c+4dx] - 12a^4b \operatorname{Sinh}[6c+4dx] - 36a^3b^2 \operatorname{Sinh}[6c+4dx] - \\
& \left. 24a^2b^3 \operatorname{Sinh}[6c+4dx] + 2a^4b \operatorname{Sinh}[4c+6dx] + 2a^3b^2 \operatorname{Sinh}[4c+6dx] + 2a^4b \operatorname{Sinh}[8c+6dx] + 2a^3b^2 \operatorname{Sinh}[8c+6dx] \right) \Big)
\end{aligned}$$

Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + dx]^3}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 114 leaves, 5 steps):

$$\frac{\sqrt{b} (3a + 5b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c+dx]}{\sqrt{b}}\right]}{2a^{7/2}d} - \frac{(a+2b) \operatorname{Cosh}[c+dx]}{a^3d} + \frac{\operatorname{Cosh}[c+dx]^3}{3a^2d} - \frac{b(a+b) \operatorname{Cosh}[c+dx]}{2a^3d(b+a \operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 861 leaves):

$$\frac{1}{1536 a^{7/2} d (a + b \operatorname{Sech}[c + dx]^2)} \left((a + 2b + a \operatorname{Cosh}[2(c + dx)])^2 \operatorname{Sech}[c + dx]^4 \left(\frac{9 a^3 \operatorname{ArcTan}\left[\frac{(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] (\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right])\right]}{\sqrt{b}}\right]}{b^{3/2}} + \right. \right.$$

$$576 a \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] (\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right]) \right)\right] + 960 b^{3/2} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] (\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right]) \right)\right] + \right.$$

$$\left. \frac{9 a^3 \operatorname{ArcTan}\left[\frac{(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] (\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right])\right]}{\sqrt{b}}\right]}{b^{3/2}} + \right.$$

$$576 a \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] (\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right]) \right)\right] + 960 b^{3/2} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] (\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right]) \right)\right] - \right.$$

$$\left. \frac{9 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \frac{9 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]}{b^{3/2}} - 96 \sqrt{a} (3a + 8b) \operatorname{Cosh}[c] \operatorname{Cosh}[dx] + \right.$$

$$32 a^{3/2} \operatorname{Cosh}[3c] \operatorname{Cosh}[3dx] - \frac{384 a^{3/2} b \operatorname{Cosh}[c+dx]}{a + 2b + a \operatorname{Cosh}[2(c+dx)]} - \frac{384 \sqrt{a} b^2 \operatorname{Cosh}[c+dx]}{a + 2b + a \operatorname{Cosh}[2(c+dx)]} - \left. \right)$$

$$288 a^{3/2} \operatorname{Sinh}[c] \operatorname{Sinh}[dx] - 768 \sqrt{a} b \operatorname{Sinh}[c] \operatorname{Sinh}[dx] + 32 a^{3/2} \operatorname{Sinh}[3c] \operatorname{Sinh}[3dx]$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]^2}{(a + b \text{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$-\frac{(a + 4 b) x}{2 a^3} + \frac{\sqrt{b} (3 a + 4 b) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{2 a^3 \sqrt{a + b} d} + \frac{\text{Cosh}[c + d x] \text{Sinh}[c + d x]}{2 a d (a + b - b \text{Tanh}[c + d x]^2)} + \frac{b \text{Tanh}[c + d x]}{a^2 d (a + b - b \text{Tanh}[c + d x]^2)}$$

Result (type 3, 791 leaves):

$$\begin{aligned}
& \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \right. \\
& \left. \left(16x + \left((a^3 - 6a^2b - 24ab^2 - 16b^3) \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] \right. \right. \right. \\
& \left. \left. \left. (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \right) / \left(b (a+b)^{3/2} d \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right) + \right. \\
& \left. \left. \frac{(a^2 + 8ab + 8b^2) \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{b (a+b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) \right) / \left(128a^2 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
& \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \left(-64(a + 2b)x + \left(-a^4 + 16a^3b + 144a^2b^2 + 256ab^3 + 128b^4 \right) \right. \right. \\
& \left. \left. \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \right) / \\
& \left(b (a+b)^{3/2} d \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right) + \frac{16a \operatorname{Cosh}[2dx] \operatorname{Sinh}[2c]}{d} + \frac{16a \operatorname{Cosh}[2c] \operatorname{Sinh}[2dx]}{d} - \\
& \left. \left. \frac{(a^3 + 18a^2b + 48ab^2 + 32b^3) \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{b (a+b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) \right) / \left(256a^3 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) - \\
& \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \left(-\frac{a \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} + \frac{\sqrt{b} (a+2b) \operatorname{Sinh}[2(c + dx)]}{(a+b) (a+2b + a \operatorname{Cosh}[2(c + dx)])} \right)}{256b^{3/2} d (a + b \operatorname{Sech}[c + dx]^2)^2} + \\
& \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \left(-\frac{(a+2b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{8b^{3/2} (a+b)^{3/2} d} + \frac{a \operatorname{Sinh}[2(c + dx)]}{8b (a+b) d (a+2b + a \operatorname{Cosh}[2(c + dx)])} \right)}{16 (a + b \operatorname{Sech}[c + dx]^2)^2}
\end{aligned}$$

Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + dx]}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$-\frac{3\sqrt{b}\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Cosh}[c+dx]}{\sqrt{b}}\right]}{2a^{5/2}d} + \frac{3\operatorname{Cosh}[c+dx]}{2a^2d} - \frac{\operatorname{Cosh}[c+dx]^3}{2ad(b+a\operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 479 leaves):

$$\frac{1}{128d(a+b\operatorname{Sech}[c+dx]^2)^2}$$

$$(a+2b+a\operatorname{Cosh}[2(c+dx)])^2\operatorname{Sech}[c+dx]^4\left(\frac{32\operatorname{Cosh}[c]\operatorname{Cosh}[dx]}{a^2} + \frac{32b\operatorname{Cosh}[c+dx]}{a^2(a+2b+a\operatorname{Cosh}[2(c+dx)])} + \frac{1}{a^{5/2}b^{3/2}}2\left(- (a^2+24b^2)\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\right.\right.\right.$$

$$\left.\left.\left(\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right)\operatorname{Sinh}[c]\operatorname{Tanh}\left[\frac{dx}{2}\right]+\operatorname{Cosh}[c]\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\operatorname{Tanh}\left[\frac{dx}{2}\right]\right)\right)\right)\right]-$$

$$a^2\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right)\operatorname{Sinh}[c]\operatorname{Tanh}\left[\frac{dx}{2}\right]+\right.\right.$$

$$\left.\left.\operatorname{Cosh}[c]\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\operatorname{Tanh}\left[\frac{dx}{2}\right]\right)\right)\right]-24b^2\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\right.$$

$$\left.\left(\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right)\operatorname{Sinh}[c]\operatorname{Tanh}\left[\frac{dx}{2}\right]+\operatorname{Cosh}[c]\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\operatorname{Tanh}\left[\frac{dx}{2}\right]\right)\right)\right]+$$

$$a^2\operatorname{ArcTan}\left[\frac{\sqrt{a}-i\sqrt{a+b}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]+a^2\operatorname{ArcTan}\left[\frac{\sqrt{a}+i\sqrt{a+b}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]+16\sqrt{a}b^{3/2}\operatorname{Sinh}[c]\operatorname{Sinh}[dx]\left.\right)\right)$$

Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]}{(a+b\operatorname{Sech}[c+dx]^2)^2} dx$$

Optimal (type 3, 99 leaves, 5 steps):

$$\frac{\sqrt{b}(3a+b)\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Cosh}[c+dx]}{\sqrt{b}}\right]}{2a^{3/2}(a+b)^2d} - \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{(a+b)^2d} - \frac{b\operatorname{Cosh}[c+dx]}{2a(a+b)d(b+a\operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 377 leaves):

$$\frac{1}{8 (a+b)^2 d (a+b \operatorname{Sech}[c+dx])^2} (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^3$$

$$\left(-\frac{2b(a+b)}{a} + \frac{1}{a^{3/2}} \sqrt{b} (3a+b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \right. \right. \right.$$

$$\left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right) (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] + \frac{1}{a^{3/2}} \sqrt{b} (3a+b)$$

$$\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right)$$

$$(a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] - 2(a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sech}[c+dx] +$$

$$2(a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sech}[c+dx] \Bigg)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]^2}{(a+b \operatorname{Sech}[c+dx])^2} dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$\frac{3\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{2(a+b)^{5/2} d} - \frac{3 \operatorname{Coth}[c+dx]}{2(a+b)^2 d} + \frac{\operatorname{Coth}[c+dx]}{2(a+b) d (a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 220 leaves):

$$\left((a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^4 \right.$$

$$\left(\left(3b \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a+2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c+dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] (a+2b+a \operatorname{Cosh}[2(c+dx)]) \right. \right.$$

$$\left. \left. (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \Bigg/ \left(\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + 2(a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Csch}[c] \operatorname{Csch}[c+dx] \operatorname{Sinh}[dx] +$$

$$b \operatorname{Sech}[2c] \operatorname{Sinh}[2dx] - \frac{b(a+2b) \operatorname{Tanh}[2c]}{a} \Bigg) \Bigg/ \left(8(a+b)^2 d (a+b \operatorname{Sech}[c+dx])^2 \right)$$

Problem 39: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^3}{(a + b \text{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$-\frac{(3a - b) \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c + d x]}{\sqrt{b}}\right]}{2 \sqrt{a} (a + b)^3 d} + \frac{(a - 3b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 (a + b)^3 d} - \frac{(a - b) \text{Cosh}[c + d x]}{2 (a + b)^2 d (b + a \text{Cosh}[c + d x]^2)} - \frac{\text{Coth}[c + d x] \text{Csch}[c + d x]}{2 (a + b) d (b + a \text{Cosh}[c + d x]^2)}$$

Result (type 3, 462 leaves):

$$\frac{1}{32 (a + b)^3 d (a + b \text{Sech}[c + d x]^2)^2} (a + 2b + a \text{Cosh}[2(c + d x)]) \text{Sech}[c + d x]^3$$

$$\left(8b(a + b) + \frac{1}{\sqrt{a}} 4\sqrt{b}(-3a + b) \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i\sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \right) \text{Sinh}[c] \text{Tanh}\left[\frac{dx}{2}\right] + \right. \right.$$

$$\left. \left. \text{Cosh}[c] \left(\sqrt{a} - i\sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right) (a + 2b + a \text{Cosh}[2(c + d x)]) \text{Sech}[c + d x] + \frac{1}{\sqrt{a}} 4\sqrt{b}(-3a + b)$$

$$\text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i\sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \text{Sinh}[c] \text{Tanh}\left[\frac{dx}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} + i\sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right)$$

$$(a + 2b + a \text{Cosh}[2(c + d x)]) \text{Sech}[c + d x] - (a + b) (a + 2b + a \text{Cosh}[2(c + d x)]) \text{Csch}\left[\frac{1}{2}(c + d x)\right]^2 \text{Sech}[c + d x] +$$

$$4(a - 3b) (a + 2b + a \text{Cosh}[2(c + d x)]) \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sech}[c + d x] -$$

$$4(a - 3b) (a + 2b + a \text{Cosh}[2(c + d x)]) \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sech}[c + d x] -$$

$$(a + b) (a + 2b + a \text{Cosh}[2(c + d x)]) \text{Sech}\left[\frac{1}{2}(c + d x)\right]^2 \text{Sech}[c + d x]$$

Problem 40: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^4}{(a + b \text{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$-\frac{(3a - 2b) \sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{2 (a + b)^{7/2} d} + \frac{(a - b) \text{Coth}[c + d x]}{(a + b)^3 d} - \frac{\text{Coth}[c + d x]^3}{3 (a + b)^2 d} - \frac{ab \text{Tanh}[c + d x]}{2 (a + b)^3 d (a + b - b \text{Tanh}[c + d x]^2)}$$

Result (type 3, 620 leaves):

$$\begin{aligned}
 & - \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Coth}[c] \operatorname{Csch}[c + dx]^2 \operatorname{Sech}[c + dx]^4}{12(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^2} + \left((3a - 2b) (a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \right. \\
 & \left. \operatorname{Sech}[c + dx]^4 \left(\left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \right. \\
 & \left. \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) \right) / \left(8\sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \\
 & \left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
 & \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \right) / \left(8\sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \Big) / \\
 & \left((a + b)^3 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Csch}[c] \operatorname{Csch}[c + dx]^3 \operatorname{Sech}[c + dx]^4 \operatorname{Sinh}[dx]}{12(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^2} + \\
 & \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Csch}[c] \operatorname{Csch}[c + dx] \operatorname{Sech}[c + dx]^4 (-a \operatorname{Sinh}[dx] + 2b \operatorname{Sinh}[dx])}{6(a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^2} + \\
 & \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c + dx]^4 (a b \operatorname{Sinh}[2c] + 2b^2 \operatorname{Sinh}[2c] - a b \operatorname{Sinh}[2dx])}{8(a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^2}
 \end{aligned}$$

Problem 41: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + dx]^4}{(a + b \operatorname{Sech}[c + dx]^2)^3} dx$$

Optimal (type 3, 242 leaves, 8 steps):

$$\begin{aligned}
 & \frac{3(a^2 + 12ab + 16b^2)x}{8a^5} - \frac{3\sqrt{b}(5a^2 + 20ab + 16b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}}\right]}{8a^5 \sqrt{a+b} d} - \frac{(5a + 8b) \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx]}{8a^2 d (a + b - b \operatorname{Tanh}[c + dx]^2)^2} + \\
 & \frac{\operatorname{Cosh}[c + dx]^3 \operatorname{Sinh}[c + dx]}{4a d (a + b - b \operatorname{Tanh}[c + dx]^2)^2} - \frac{b(7a + 12b) \operatorname{Tanh}[c + dx]}{8a^3 d (a + b - b \operatorname{Tanh}[c + dx]^2)^2} - \frac{3b(a + 2b) \operatorname{Tanh}[c + dx]}{2a^4 d (a + b - b \operatorname{Tanh}[c + dx]^2)^2}
 \end{aligned}$$

Result (type 3, 4019 leaves):

$$\left(3(a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \right)$$

$$\begin{aligned}
& \left(\frac{(3a^2 + 8ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right] - a\sqrt{b} (3a^2 + 16ab + 16b^2 + 3a(a+2b) \operatorname{Cosh}[2(c+dx)]) \operatorname{Sinh}[2(c+dx)]}{(a+b)^{5/2} (a+b)^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])^2} \right) \Bigg/ \\
& (16384b^{5/2}d(a+b \operatorname{Sech}[c+dx]^2)^3) + \left((a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \right. \\
& \left. - \frac{3a(a+2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right] + \sqrt{b} (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \operatorname{Cosh}[2(c+dx)]) \operatorname{Sinh}[2(c+dx)]}{(a+b)^{5/2} (a+b)^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])^2} \right) \Bigg/ \\
& (16384b^{5/2}d(a+b \operatorname{Sech}[c+dx]^2)^3) - \frac{1}{512(a+b \operatorname{Sech}[c+dx]^2)^3} \\
& 3(a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \left(\frac{1}{(a+b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \right. \\
& \left. \left(\left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \\
& \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) \Bigg/ (64a^3b^2\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}) - \\
& \left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
& \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \Bigg/ (64a^3b^2\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}) \Bigg) + \\
& \frac{1}{128a^3b^2(a+b)^2d(a+2b+a \operatorname{Cosh}[2c+2dx])^2} \operatorname{Sech}[2c] (768a^4b^2dx \operatorname{Cosh}[2c] + 3584a^3b^3dx \operatorname{Cosh}[2c] + 6912a^2b^4dx \operatorname{Cosh}[2c] + \\
& 6144ab^5dx \operatorname{Cosh}[2c] + 2048b^6dx \operatorname{Cosh}[2c] + 512a^4b^2dx \operatorname{Cosh}[2dx] + 2048a^3b^3dx \operatorname{Cosh}[2dx] + 2560a^2b^4dx \operatorname{Cosh}[2dx] + \\
& 1024ab^5dx \operatorname{Cosh}[2dx] + 512a^4b^2dx \operatorname{Cosh}[4c+2dx] + 2048a^3b^3dx \operatorname{Cosh}[4c+2dx] + 2560a^2b^4dx \operatorname{Cosh}[4c+2dx] + \\
& 1024ab^5dx \operatorname{Cosh}[4c+2dx] + 128a^4b^2dx \operatorname{Cosh}[2c+4dx] + 256a^3b^3dx \operatorname{Cosh}[2c+4dx] + 128a^2b^4dx \operatorname{Cosh}[2c+4dx] + \\
& 128a^4b^2dx \operatorname{Cosh}[6c+4dx] + 256a^3b^3dx \operatorname{Cosh}[6c+4dx] + 128a^2b^4dx \operatorname{Cosh}[6c+4dx] - 9a^6 \operatorname{Sinh}[2c] + 12a^5b \operatorname{Sinh}[2c] + \\
& 684a^4b^2 \operatorname{Sinh}[2c] + 2880a^3b^3 \operatorname{Sinh}[2c] + 5280a^2b^4 \operatorname{Sinh}[2c] + 4608ab^5 \operatorname{Sinh}[2c] + 1536b^6 \operatorname{Sinh}[2c] + 9a^6 \operatorname{Sinh}[2dx] - \\
& 14a^5b \operatorname{Sinh}[2dx] - 608a^4b^2 \operatorname{Sinh}[2dx] - 2112a^3b^3 \operatorname{Sinh}[2dx] - 2560a^2b^4 \operatorname{Sinh}[2dx] - 1024ab^5 \operatorname{Sinh}[2dx] - 3a^6 \operatorname{Sinh}[4c+2dx] + \\
& 10a^5b \operatorname{Sinh}[4c+2dx] + 304a^4b^2 \operatorname{Sinh}[4c+2dx] + 1056a^3b^3 \operatorname{Sinh}[4c+2dx] + 1280a^2b^4 \operatorname{Sinh}[4c+2dx] + 512ab^5 \operatorname{Sinh}[4c+2dx] + \\
& 3a^6 \operatorname{Sinh}[2c+4dx] - 12a^5b \operatorname{Sinh}[2c+4dx] - 204a^4b^2 \operatorname{Sinh}[2c+4dx] - 384a^3b^3 \operatorname{Sinh}[2c+4dx] - 192a^2b^4 \operatorname{Sinh}[2c+4dx]) \Bigg) + \\
& \frac{1}{512(a+b \operatorname{Sech}[c+dx]^2)^3} (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \left(\frac{12(7a^2 + 32ab + 32b^2)x}{a^5} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(a+b)^2} (a^7 - 14 a^6 b + 336 a^5 b^2 + 5600 a^4 b^3 + 22400 a^3 b^4 + 37632 a^2 b^5 + 28672 a b^6 + 8192 b^7) \\
& \left(\left(3 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \right. \\
& \quad \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \operatorname{Cosh}[2 c] \right) / \left(64 a^5 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) - \right. \\
& \quad \left. \left(3 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \right. \\
& \quad \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \operatorname{Sinh}[2 c] \right) / \left(64 a^5 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) \right) + \\
& \frac{1}{16 a^5 b (a+b) d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^2} \operatorname{Sech}[2 c] (-a^6 \operatorname{Sinh}[2 c] - 52 a^5 b \operatorname{Sinh}[2 c] - 500 a^4 b^2 \operatorname{Sinh}[2 c] - \\
& \quad 1920 a^3 b^3 \operatorname{Sinh}[2 c] - 3520 a^2 b^4 \operatorname{Sinh}[2 c] - 3072 a b^5 \operatorname{Sinh}[2 c] - 1024 b^6 \operatorname{Sinh}[2 c] + a^6 \operatorname{Sinh}[2 d x] + \\
& \quad 50 a^5 b \operatorname{Sinh}[2 d x] + 400 a^4 b^2 \operatorname{Sinh}[2 d x] + 1120 a^3 b^3 \operatorname{Sinh}[2 d x] + 1280 a^2 b^4 \operatorname{Sinh}[2 d x] + 512 a b^5 \operatorname{Sinh}[2 d x]) + \\
& \frac{1}{64 a^5 b^2 (a+b)^2 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])} \operatorname{Sech}[2 c] (-3 a^7 \operatorname{Sinh}[2 c] + 42 a^6 b \operatorname{Sinh}[2 c] + 2192 a^5 b^2 \operatorname{Sinh}[2 c] + 16480 a^4 b^3 \operatorname{Sinh}[2 c] + \\
& \quad 51200 a^3 b^4 \operatorname{Sinh}[2 c] + 77824 a^2 b^5 \operatorname{Sinh}[2 c] + 57344 a b^6 \operatorname{Sinh}[2 c] + 16384 b^7 \operatorname{Sinh}[2 c] + 3 a^7 \operatorname{Sinh}[2 d x] - 44 a^6 b \operatorname{Sinh}[2 d x] - \\
& \quad 1900 a^5 b^2 \operatorname{Sinh}[2 d x] - 10880 a^4 b^3 \operatorname{Sinh}[2 d x] - 23360 a^3 b^4 \operatorname{Sinh}[2 d x] - 21504 a^2 b^5 \operatorname{Sinh}[2 d x] - 7168 a b^6 \operatorname{Sinh}[2 d x]) + \\
& (a+2 b) \left(-\frac{12 \operatorname{Cosh}[2 c+2 d x]}{a^4 d} - \frac{12 \operatorname{Sinh}[2 c+2 d x]}{a^4 d} \right) + (a+2 b) \left(\frac{12 \operatorname{Cosh}[2 c+2 d x]}{a^4 d} - \frac{12 \operatorname{Sinh}[2 c+2 d x]}{a^4 d} \right) + \\
& \left. \frac{2 \operatorname{Sinh}[4 c+4 d x]}{a^3 d} \right) + \\
& \frac{1}{256 (a+b \operatorname{Sech}[c+d x])^2} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \\
& \operatorname{Sech}[c+d x]^6 \\
& \left(\frac{1}{(a+b)^2} (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right. \\
& \quad \left. - \left(\left(3 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \right. \right. \\
& \quad \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \operatorname{Cosh}[2 c] \right) / \left(64 a^4 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) \right) + \\
& \quad \left. \left(3 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \right. \\
& \quad \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \operatorname{Sinh}[2 c] \right) / \left(64 a^4 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) \right) +
\end{aligned}$$

$$\left. \left. \begin{aligned} & (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \operatorname{Sinh}[2c] \right) \Bigg/ \left(64a^4 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \Bigg) + \\ & \frac{1}{128 a^4 b^2 (a+b)^2 d (a+2b+a \operatorname{Cosh}[2c+2dx])^2} \operatorname{Sech}[2c] \left(-4608 a^5 b^2 dx \operatorname{Cosh}[2c] - 30720 a^4 b^3 dx \operatorname{Cosh}[2c] - \right. \\ & 84480 a^3 b^4 dx \operatorname{Cosh}[2c] - 119808 a^2 b^5 dx \operatorname{Cosh}[2c] - 86016 a b^6 dx \operatorname{Cosh}[2c] - 24576 b^7 dx \operatorname{Cosh}[2c] - 3072 a^5 b^2 dx \operatorname{Cosh}[2dx] - \\ & 18432 a^4 b^3 dx \operatorname{Cosh}[2dx] - 39936 a^3 b^4 dx \operatorname{Cosh}[2dx] - 36864 a^2 b^5 dx \operatorname{Cosh}[2dx] - 12288 a b^6 dx \operatorname{Cosh}[2dx] - \\ & 3072 a^5 b^2 dx \operatorname{Cosh}[4c+2dx] - 18432 a^4 b^3 dx \operatorname{Cosh}[4c+2dx] - 39936 a^3 b^4 dx \operatorname{Cosh}[4c+2dx] - 36864 a^2 b^5 dx \operatorname{Cosh}[4c+2dx] - \\ & 12288 a b^6 dx \operatorname{Cosh}[4c+2dx] - 768 a^5 b^2 dx \operatorname{Cosh}[2c+4dx] - 3072 a^4 b^3 dx \operatorname{Cosh}[2c+4dx] - 3840 a^3 b^4 dx \operatorname{Cosh}[2c+4dx] - \\ & 1536 a^2 b^5 dx \operatorname{Cosh}[2c+4dx] - 768 a^5 b^2 dx \operatorname{Cosh}[6c+4dx] - 3072 a^4 b^3 dx \operatorname{Cosh}[6c+4dx] - 3840 a^3 b^4 dx \operatorname{Cosh}[6c+4dx] - \\ & 1536 a^2 b^5 dx \operatorname{Cosh}[6c+4dx] + 9 a^7 \operatorname{Sinh}[2c] - 54 a^6 b \operatorname{Sinh}[2c] - 2392 a^5 b^2 \operatorname{Sinh}[2c] - 13968 a^4 b^3 \operatorname{Sinh}[2c] - \\ & 36480 a^3 b^4 \operatorname{Sinh}[2c] - 50432 a^2 b^5 \operatorname{Sinh}[2c] - 35840 a b^6 \operatorname{Sinh}[2c] - 10240 b^7 \operatorname{Sinh}[2c] - 9 a^7 \operatorname{Sinh}[2dx] + 56 a^6 b \operatorname{Sinh}[2dx] + \\ & 2552 a^5 b^2 \operatorname{Sinh}[2dx] + 13184 a^4 b^3 \operatorname{Sinh}[2dx] + 27072 a^3 b^4 \operatorname{Sinh}[2dx] + 24576 a^2 b^5 \operatorname{Sinh}[2dx] + 8192 a b^6 \operatorname{Sinh}[2dx] + \\ & 3 a^7 \operatorname{Sinh}[4c+2dx] - 24 a^6 b \operatorname{Sinh}[4c+2dx] - 600 a^5 b^2 \operatorname{Sinh}[4c+2dx] - 3200 a^4 b^3 \operatorname{Sinh}[4c+2dx] - 6720 a^3 b^4 \operatorname{Sinh}[4c+2dx] - \\ & 6144 a^2 b^5 \operatorname{Sinh}[4c+2dx] - 2048 a b^6 \operatorname{Sinh}[4c+2dx] - 3 a^7 \operatorname{Sinh}[2c+4dx] + 26 a^6 b \operatorname{Sinh}[2c+4dx] + 992 a^5 b^2 \operatorname{Sinh}[2c+4dx] + \\ & 3648 a^4 b^3 \operatorname{Sinh}[2c+4dx] + 4480 a^3 b^4 \operatorname{Sinh}[2c+4dx] + 1792 a^2 b^5 \operatorname{Sinh}[2c+4dx] + 256 a^5 b^2 \operatorname{Sinh}[6c+4dx] + \\ & 1024 a^4 b^3 \operatorname{Sinh}[6c+4dx] + 1280 a^3 b^4 \operatorname{Sinh}[6c+4dx] + 512 a^2 b^5 \operatorname{Sinh}[6c+4dx] + 64 a^5 b^2 \operatorname{Sinh}[4c+6dx] + \\ & 128 a^4 b^3 \operatorname{Sinh}[4c+6dx] + 64 a^3 b^4 \operatorname{Sinh}[4c+6dx] + 64 a^5 b^2 \operatorname{Sinh}[8c+6dx] + 128 a^4 b^3 \operatorname{Sinh}[8c+6dx] + 64 a^3 b^4 \operatorname{Sinh}[8c+6dx] \Bigg) - \\ & \frac{1}{8192 b^2 (a+b)^2 d (a+b \operatorname{Sech}[c+dx])^2} (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \\ & \operatorname{Sech}[c+dx]^6 \left(\frac{6 a^2 \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c]-\operatorname{Sinh}[2c]) ((a+2b) \operatorname{Sinh}[dx]-a \operatorname{Sinh}[2c+dx])}{2 \sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4}} \right] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c])}{\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} + \right. \\ & (a \operatorname{Sech}[2c] \left((-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \operatorname{Sinh}[2dx] + \right. \\ & \left. a (-3 a^3 + 2 a^2 b + 24 a b^2 + 16 b^3) \operatorname{Sinh}[2(c+2dx)] + (3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4) \operatorname{Sinh}[4c+2dx] \right) + \\ & \left. (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \operatorname{Tanh}[2c] \right) \Bigg/ \left(a^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])^2 \right) \Bigg) \Bigg) -
\end{aligned}$$

Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c+dx]^3}{(a+b \operatorname{Sech}[c+dx])^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{5\sqrt{b} (3a + 7b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c+dx]}{\sqrt{b}}\right]}{8a^{9/2}d} - \frac{(a+3b) \operatorname{Cosh}[c+dx]}{a^4d} + \frac{\operatorname{Cosh}[c+dx]^3}{3a^3d} + \frac{b^2(a+b) \operatorname{Cosh}[c+dx]}{4a^4d(b+a \operatorname{Cosh}[c+dx]^2)} - \frac{b(9a+13b) \operatorname{Cosh}[c+dx]}{8a^4d(b+a \operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 1364 leaves):

$$\begin{aligned} & - \left(\left(\left(\frac{3 \left(\operatorname{ArcTan}\left[\frac{\sqrt{a} - i\sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{a} + i\sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]}{\sqrt{a}} \right) + \frac{2\sqrt{b} \operatorname{Cosh}[c+dx] (3a + 10b + 3a \operatorname{Cosh}[2(c+dx)])}{(a + 2b + a \operatorname{Cosh}[2(c+dx)])^2} \right) \right) \right. \\ & \left. \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c+dx]^6 \right) / \left(8192 b^{5/2} d (a + b \operatorname{Sech}[c+dx]^2)^3 \right) \right) - \\ & \frac{1}{2048 a^{3/2} b^{5/2} d (a + b \operatorname{Sech}[c+dx]^2)^3} \left(- (3a - 4b) \left(\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \right. \right. \right. \right. \\ & \left. \left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right) + \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \right. \\ & \left. \left. \left((\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) - \\ & \left. \frac{2\sqrt{a}\sqrt{b} \operatorname{Cosh}[c+dx] (3a^2 + 6ab + 8b^2 + a(3a - 4b) \operatorname{Cosh}[2(c+dx)])}{(a + 2b + a \operatorname{Cosh}[2(c+dx)])^2} \right) (a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c+dx]^6 + \\ & \frac{1}{49152 a^{9/2} b^{5/2} d (a + b \operatorname{Sech}[c+dx]^2)^3} \left(3 (3a^4 - 40a^3b + 720a^2b^2 + 6720ab^3 + 8960b^4) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \right. \\ & \left. \left. \left((\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) + \\ & 3 (3a^4 - 40a^3b + 720a^2b^2 + 6720ab^3 + 8960b^4) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \\ & \left. \left. \left((\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) + \\ & \left. \left(2\sqrt{a}\sqrt{b} \operatorname{Cosh}[c+dx] (9a^5 - 90a^4b - 10144a^3b^2 - 48672a^2b^3 - 85120ab^4 - 53760b^5 + a(9a^4 - 120a^3b - 12432a^2b^2 - 47936ab^3 - \right. \right. \\ & \left. \left. 44800b^4) \operatorname{Cosh}[2(c+dx)] - 128a^2b^2(15a + 28b) \operatorname{Cosh}[4(c+dx)] + 128a^3b^2 \operatorname{Cosh}[6(c+dx)] \right) \right) / (a + 2b + a \operatorname{Cosh}[2(c+dx)])^2 \right) \\ & (a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c+dx]^6 + \frac{1}{16384 a^{7/2} d (a + b \operatorname{Sech}[c+dx]^2)^3} 3 (a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \\ & \operatorname{Sech}[c+dx]^6 \end{aligned}$$

$$\left(-\frac{1}{b^{5/2}} 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] - \frac{1}{b^{5/2}} 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \right] + 512 \sqrt{a} \operatorname{Cosh}[c] \operatorname{Cosh}[d x] - \frac{8 \sqrt{a} (a^3 + 24 a^2 b + 80 a b^2 + 64 b^3) \operatorname{Cosh}[c + d x]}{b (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2} - \frac{2 \sqrt{a} (3 a^3 - 24 a^2 b - 400 a b^2 - 576 b^3) \operatorname{Cosh}[c + d x]}{b^2 (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])} + 512 \sqrt{a} \operatorname{Sinh}[c] \operatorname{Sinh}[d x] \right)$$

Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + d x]^2}{(a + b \operatorname{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$-\frac{(a + 6 b) x}{2 a^4} + \frac{\sqrt{b} (15 a^2 + 40 a b + 24 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{8 a^4 (a + b)^{3/2} d} + \frac{\operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d (a + b - b \operatorname{Tanh}[c + d x]^2)^2} + \frac{3 b \operatorname{Tanh}[c + d x]}{4 a^2 d (a + b - b \operatorname{Tanh}[c + d x]^2)^2} + \frac{b (11 a + 12 b) \operatorname{Tanh}[c + d x]}{8 a^3 (a + b) d (a + b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 3106 leaves):

$$-\left(\left(5 (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \operatorname{Sech}[c + d x]^6 \left(\frac{(3 a^2 + 8 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{(a + b)^{5/2}} - \frac{a \sqrt{b} (3 a^2 + 16 a b + 16 b^2 + 3 a (a + 2 b) \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sinh}[2 (c + d x)]}{(a + b)^2 (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2} \right) \right) / (8192 b^{5/2} d (a + b \operatorname{Sech}[c + d x]^2)^3) \right) - \left((a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \operatorname{Sech}[c + d x]^6 \right)$$

$$\left(-\frac{3 a (a+2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{\sqrt{b} (3 a^3+14 a^2 b+24 a b^2+16 b^3+a (3 a^2+4 a b+4 b^2) \operatorname{Cosh}[2(c+d x)]) \operatorname{Sinh}[2(c+d x)]}{(a+b)^2 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2} \right) \Bigg/$$

$$(2048 b^{5/2} d (a+b \operatorname{Sech}[c+d x]^2)^3) + \frac{1}{32 (a+b \operatorname{Sech}[c+d x]^2)^3}$$

$$(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6 \left(\frac{1}{(a+b)^2} (3 a^5-10 a^4 b+80 a^3 b^2+480 a^2 b^3+640 a b^4+256 b^5) \right.$$

$$\left. \left(\left(i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right. \right. \right.$$

$$\left. \left. \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right] \operatorname{Cosh}[2 c] \right) \Bigg/ (64 a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}) - \right.$$

$$\left. \left(i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right. \right.$$

$$\left. \left. \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right] \operatorname{Sinh}[2 c] \right) \Bigg/ (64 a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}) \right) +$$

$$\frac{1}{128 a^3 b^2 (a+b)^2 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^2} \operatorname{Sech}[2 c] (768 a^4 b^2 d x \operatorname{Cosh}[2 c]+3584 a^3 b^3 d x \operatorname{Cosh}[2 c]+6912 a^2 b^4 d x \operatorname{Cosh}[2 c]+$$

$$6144 a b^5 d x \operatorname{Cosh}[2 c]+2048 b^6 d x \operatorname{Cosh}[2 c]+512 a^4 b^2 d x \operatorname{Cosh}[2 d x]+2048 a^3 b^3 d x \operatorname{Cosh}[2 d x]+2560 a^2 b^4 d x \operatorname{Cosh}[2 d x]+$$

$$1024 a b^5 d x \operatorname{Cosh}[2 d x]+512 a^4 b^2 d x \operatorname{Cosh}[4 c+2 d x]+2048 a^3 b^3 d x \operatorname{Cosh}[4 c+2 d x]+2560 a^2 b^4 d x \operatorname{Cosh}[4 c+2 d x]+$$

$$1024 a b^5 d x \operatorname{Cosh}[4 c+2 d x]+128 a^4 b^2 d x \operatorname{Cosh}[2 c+4 d x]+256 a^3 b^3 d x \operatorname{Cosh}[2 c+4 d x]+128 a^2 b^4 d x \operatorname{Cosh}[2 c+4 d x]+$$

$$128 a^4 b^2 d x \operatorname{Cosh}[6 c+4 d x]+256 a^3 b^3 d x \operatorname{Cosh}[6 c+4 d x]+128 a^2 b^4 d x \operatorname{Cosh}[6 c+4 d x]-9 a^6 \operatorname{Sinh}[2 c]+12 a^5 b \operatorname{Sinh}[2 c]+$$

$$684 a^4 b^2 \operatorname{Sinh}[2 c]+2880 a^3 b^3 \operatorname{Sinh}[2 c]+5280 a^2 b^4 \operatorname{Sinh}[2 c]+4608 a b^5 \operatorname{Sinh}[2 c]+1536 b^6 \operatorname{Sinh}[2 c]+9 a^6 \operatorname{Sinh}[2 d x]-$$

$$14 a^5 b \operatorname{Sinh}[2 d x]-608 a^4 b^2 \operatorname{Sinh}[2 d x]-2112 a^3 b^3 \operatorname{Sinh}[2 d x]-2560 a^2 b^4 \operatorname{Sinh}[2 d x]-1024 a b^5 \operatorname{Sinh}[2 d x]-3 a^6 \operatorname{Sinh}[4 c+2 d x]+$$

$$10 a^5 b \operatorname{Sinh}[4 c+2 d x]+304 a^4 b^2 \operatorname{Sinh}[4 c+2 d x]+1056 a^3 b^3 \operatorname{Sinh}[4 c+2 d x]+1280 a^2 b^4 \operatorname{Sinh}[4 c+2 d x]+512 a b^5 \operatorname{Sinh}[4 c+2 d x]+$$

$$3 a^6 \operatorname{Sinh}[2 c+4 d x]-12 a^5 b \operatorname{Sinh}[2 c+4 d x]-204 a^4 b^2 \operatorname{Sinh}[2 c+4 d x]-384 a^3 b^3 \operatorname{Sinh}[2 c+4 d x]-192 a^2 b^4 \operatorname{Sinh}[2 c+4 d x]) \Bigg) +$$

$$\frac{1}{128 (a+b \operatorname{Sech}[c+d x]^2)^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6$$

$$\left(\frac{1}{(a+b)^2} (a^6-8 a^5 b+120 a^4 b^2+1280 a^3 b^3+3200 a^2 b^4+3072 a b^5+1024 b^6) \right.$$

$$\left. \left(-\left(\left(3 i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right. \right. \right. \right.$$

$$\left. \left. \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right] \operatorname{Cosh}[2 c] \right) \Bigg/ (64 a^4 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}) \right) +$$

$$\begin{aligned}
& \left(3 i \operatorname{ArcTan}[\operatorname{Sech}[d x] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right. \\
& \quad \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right] \operatorname{Sinh}[2 c] \Big/ \left(64 a^4 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) \Big) + \\
& \frac{1}{128 a^4 b^2 (a+b)^2 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^2} \operatorname{Sech}[2 c] \left(-4608 a^5 b^2 d x \operatorname{Cosh}[2 c]-30720 a^4 b^3 d x \operatorname{Cosh}[2 c]- \right. \\
& \quad 84480 a^3 b^4 d x \operatorname{Cosh}[2 c]-119808 a^2 b^5 d x \operatorname{Cosh}[2 c]-86016 a b^6 d x \operatorname{Cosh}[2 c]-24576 b^7 d x \operatorname{Cosh}[2 c]-3072 a^5 b^2 d x \operatorname{Cosh}[2 d x]- \\
& \quad 18432 a^4 b^3 d x \operatorname{Cosh}[2 d x]-39936 a^3 b^4 d x \operatorname{Cosh}[2 d x]-36864 a^2 b^5 d x \operatorname{Cosh}[2 d x]-12288 a b^6 d x \operatorname{Cosh}[2 d x]- \\
& \quad 3072 a^5 b^2 d x \operatorname{Cosh}[4 c+2 d x]-18432 a^4 b^3 d x \operatorname{Cosh}[4 c+2 d x]-39936 a^3 b^4 d x \operatorname{Cosh}[4 c+2 d x]-36864 a^2 b^5 d x \operatorname{Cosh}[4 c+2 d x]- \\
& \quad 12288 a b^6 d x \operatorname{Cosh}[4 c+2 d x]-768 a^5 b^2 d x \operatorname{Cosh}[2 c+4 d x]-3072 a^4 b^3 d x \operatorname{Cosh}[2 c+4 d x]-3840 a^3 b^4 d x \operatorname{Cosh}[2 c+4 d x]- \\
& \quad 1536 a^2 b^5 d x \operatorname{Cosh}[2 c+4 d x]-768 a^5 b^2 d x \operatorname{Cosh}[6 c+4 d x]-3072 a^4 b^3 d x \operatorname{Cosh}[6 c+4 d x]-3840 a^3 b^4 d x \operatorname{Cosh}[6 c+4 d x]- \\
& \quad 1536 a^2 b^5 d x \operatorname{Cosh}[6 c+4 d x]+9 a^7 \operatorname{Sinh}[2 c]-54 a^6 b \operatorname{Sinh}[2 c]-2392 a^5 b^2 \operatorname{Sinh}[2 c]-13968 a^4 b^3 \operatorname{Sinh}[2 c]- \\
& \quad 36480 a^3 b^4 \operatorname{Sinh}[2 c]-50432 a^2 b^5 \operatorname{Sinh}[2 c]-35840 a b^6 \operatorname{Sinh}[2 c]-10240 b^7 \operatorname{Sinh}[2 c]-9 a^7 \operatorname{Sinh}[2 d x]+56 a^6 b \operatorname{Sinh}[2 d x]+ \\
& \quad 2552 a^5 b^2 \operatorname{Sinh}[2 d x]+13184 a^4 b^3 \operatorname{Sinh}[2 d x]+27072 a^3 b^4 \operatorname{Sinh}[2 d x]+24576 a^2 b^5 \operatorname{Sinh}[2 d x]+8192 a b^6 \operatorname{Sinh}[2 d x]+ \\
& \quad 3 a^7 \operatorname{Sinh}[4 c+2 d x]-24 a^6 b \operatorname{Sinh}[4 c+2 d x]-600 a^5 b^2 \operatorname{Sinh}[4 c+2 d x]-3200 a^4 b^3 \operatorname{Sinh}[4 c+2 d x]-6720 a^3 b^4 \operatorname{Sinh}[4 c+2 d x]- \\
& \quad 6144 a^2 b^5 \operatorname{Sinh}[4 c+2 d x]-2048 a b^6 \operatorname{Sinh}[4 c+2 d x]-3 a^7 \operatorname{Sinh}[2 c+4 d x]+26 a^6 b \operatorname{Sinh}[2 c+4 d x]+992 a^5 b^2 \operatorname{Sinh}[2 c+4 d x]+ \\
& \quad 3648 a^4 b^3 \operatorname{Sinh}[2 c+4 d x]+4480 a^3 b^4 \operatorname{Sinh}[2 c+4 d x]+1792 a^2 b^5 \operatorname{Sinh}[2 c+4 d x]+256 a^5 b^2 \operatorname{Sinh}[6 c+4 d x]+ \\
& \quad 1024 a^4 b^3 \operatorname{Sinh}[6 c+4 d x]+1280 a^3 b^4 \operatorname{Sinh}[6 c+4 d x]+512 a^2 b^5 \operatorname{Sinh}[6 c+4 d x]+64 a^5 b^2 \operatorname{Sinh}[4 c+6 d x]+ \\
& \quad 128 a^4 b^3 \operatorname{Sinh}[4 c+6 d x]+64 a^3 b^4 \operatorname{Sinh}[4 c+6 d x]+64 a^5 b^2 \operatorname{Sinh}[8 c+6 d x]+128 a^4 b^3 \operatorname{Sinh}[8 c+6 d x]+64 a^3 b^4 \operatorname{Sinh}[8 c+6 d x] \Big) + \\
& \frac{1}{4096 b^2 (a+b)^2 d (a+b \operatorname{Sech}[c+d x])^2} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \\
& \operatorname{Sech}[c+d x]^6 \\
& \left(\frac{6 a^2 \operatorname{ArcTan} \left[\frac{\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c]) ((a+2 b) \operatorname{Sinh}[d x]-a \operatorname{Sinh}[2 c+d x])}{2 \sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4}} \right] (\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c])}{\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4}} + \right. \\
& \quad \left. (a \operatorname{Sech}[2 c] \left((-9 a^4-16 a^3 b+48 a^2 b^2+128 a b^3+64 b^4) \operatorname{Sinh}[2 d x]+ \right. \right. \\
& \quad \left. \left. a (-3 a^3+2 a^2 b+24 a b^2+16 b^3) \operatorname{Sinh}[2(c+2 d x)]+(3 a^4-64 a^2 b^2-128 a b^3-64 b^4) \operatorname{Sinh}[4 c+2 d x] \right) + \right. \\
& \quad \left. \left. (9 a^5+18 a^4 b-64 a^3 b^2-256 a^2 b^3-320 a b^4-128 b^5) \operatorname{Tanh}[2 c] \right) \Big/ \left(a^2 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \right) \Big) \Big)
\end{aligned}$$

Problem 44: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c+d x]}{(a+b \operatorname{Sech}[c+d x])^2} dx$$

Optimal (type 3, 116 leaves, 5 steps):

$$-\frac{15\sqrt{b}\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Cosh}[c+dx]}{\sqrt{b}}\right]}{8a^{7/2}d} + \frac{15\operatorname{Cosh}[c+dx]}{8a^3d} - \frac{\operatorname{Cosh}[c+dx]^5}{4ad(b+a\operatorname{Cosh}[c+dx]^2)^2} - \frac{5\operatorname{Cosh}[c+dx]^3}{8a^2d(b+a\operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 1272 leaves):

$$\frac{1}{4096a^{5/2}b^{5/2}d(a+b\operatorname{Sech}[c+dx]^2)^3} \left(3(a^2-4ab+16b^2)\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right)\operatorname{Sinh}[c]\operatorname{Tanh}\left[\frac{dx}{2}\right]+ \right.\right.\right. \\ \left.\left.\left.\operatorname{Cosh}[c]\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\operatorname{Tanh}\left[\frac{dx}{2}\right]\right)\right)\right]+3(a^2-4ab+16b^2)\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\right.\right. \\ \left.\left.\left(\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right)\operatorname{Sinh}[c]\operatorname{Tanh}\left[\frac{dx}{2}\right]+ \operatorname{Cosh}[c]\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\operatorname{Tanh}\left[\frac{dx}{2}\right]\right)\right)\right]\right) + \\ \frac{8\sqrt{a}b^{3/2}(a^2+12ab+16b^2)\operatorname{Cosh}[c+dx]}{(a+2b+a\operatorname{Cosh}[2(c+dx)])^2} + \frac{2\sqrt{a}\sqrt{b}(3a^2-12ab-80b^2)\operatorname{Cosh}[c+dx]}{a+2b+a\operatorname{Cosh}[2(c+dx)]} \Big) (a+2b+a\operatorname{Cosh}[2c+2dx])^3\operatorname{Sech}[c+dx]^6 + \\ \left(5 \left(\frac{3\left(\operatorname{ArcTan}\left[\frac{\sqrt{a}-i\sqrt{a+b}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)}\right]}{\sqrt{b}}\right)+\operatorname{ArcTan}\left[\frac{\sqrt{a}+i\sqrt{a+b}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)}\right]}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{2\sqrt{b}\operatorname{Cosh}[c+dx](3a+10b+3a\operatorname{Cosh}[2(c+dx)])}{(a+2b+a\operatorname{Cosh}[2(c+dx)])^2} \right) \right) \\ (a+2b+a\operatorname{Cosh}[2c+2dx])^3\operatorname{Sech}[c+dx]^6 \Big) / \left(4096b^{5/2}d(a+b\operatorname{Sech}[c+dx]^2)^3 \right) + \\ \frac{1}{4096a^{3/2}b^{5/2}d(a+b\operatorname{Sech}[c+dx]^2)^3} \left(-(3a-4b)\left(\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right)\operatorname{Sinh}[c]\operatorname{Tanh}\left[\frac{dx}{2}\right]+ \right.\right.\right. \right. \\ \left.\left.\left.\operatorname{Cosh}[c]\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\operatorname{Tanh}\left[\frac{dx}{2}\right]\right)\right)\right]\right)+\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\right.\right. \\ \left.\left.\left(\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right)\operatorname{Sinh}[c]\operatorname{Tanh}\left[\frac{dx}{2}\right]+ \operatorname{Cosh}[c]\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\operatorname{Tanh}\left[\frac{dx}{2}\right]\right)\right)\right]\right) \Big) - \\ \frac{2\sqrt{a}\sqrt{b}\operatorname{Cosh}[c+dx](3a^2+6ab+8b^2+a(3a-4b)\operatorname{Cosh}[2(c+dx)])}{(a+2b+a\operatorname{Cosh}[2(c+dx)])^2} \Big) (a+2b+a\operatorname{Cosh}[2c+2dx])^3\operatorname{Sech}[c+dx]^6 + \\ \frac{1}{4096a^{7/2}d(a+b\operatorname{Sech}[c+dx]^2)^3} (a+2b+a\operatorname{Cosh}[2c+2dx])^3\operatorname{Sech}[c+dx]^6$$

$$\left(-\frac{1}{b^{5/2}} 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] - \frac{1}{b^{5/2}} 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) + 512 \sqrt{a} \operatorname{Cosh}[c] \operatorname{Cosh}[dx] - \frac{8 \sqrt{a} (a^3 + 24 a^2 b + 80 a b^2 + 64 b^3) \operatorname{Cosh}[c + dx]}{b (a + 2 b + a \operatorname{Cosh}[2 (c + dx)])^2} - \frac{2 \sqrt{a} (3 a^3 - 24 a^2 b - 400 a b^2 - 576 b^3) \operatorname{Cosh}[c + dx]}{b^2 (a + 2 b + a \operatorname{Cosh}[2 (c + dx)])} + 512 \sqrt{a} \operatorname{Sinh}[c] \operatorname{Sinh}[dx] \right)$$

Problem 45: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c + dx]}{(a + b \operatorname{Sech}[c + dx]^2)^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{\sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c + dx]}{\sqrt{b}}\right]}{8 a^{5/2} (a + b)^3 d} - \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]}{(a + b)^3 d} - \frac{b \operatorname{Cosh}[c + dx]^3}{4 a (a + b) d (b + a \operatorname{Cosh}[c + dx]^2)^2} - \frac{b (7 a + 3 b) \operatorname{Cosh}[c + dx]}{8 a^2 (a + b)^2 d (b + a \operatorname{Cosh}[c + dx]^2)}$$

Result (type 3, 440 leaves):

$$\frac{1}{64 (a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^3} \left((a + 2 b + a \operatorname{Cosh}[2 (c + dx)]) \operatorname{Sech}[c + dx]^5 \left(\frac{8 b^2 (a + b)^2}{a^2} - \frac{2 b (a + b) (9 a + 5 b) (a + 2 b + a \operatorname{Cosh}[2 (c + dx)])}{a^2} + \frac{1}{a^{5/2}} \sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) \right. \\ \left. (a + 2 b + a \operatorname{Cosh}[2 (c + dx)])^2 \operatorname{Sech}[c + dx] + \frac{1}{a^{5/2}} \sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) \right. \\ \left. (a + 2 b + a \operatorname{Cosh}[2 (c + dx)])^2 \operatorname{Sech}[c + dx] - 8 (a + 2 b + a \operatorname{Cosh}[2 (c + dx)])^2 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2} (c + dx)\right]\right] \operatorname{Sech}[c + dx] + \right. \\ \left. 8 (a + 2 b + a \operatorname{Cosh}[2 (c + dx)])^2 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} (c + dx)\right]\right] \operatorname{Sech}[c + dx] \right)$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^2}{(a + b \text{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{15 \sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{8 (a + b)^{7/2} d} - \frac{15 \text{Coth}[c + d x]}{8 (a + b)^3 d} + \frac{\text{Coth}[c + d x]}{4 (a + b) d (a + b - b \text{Tanh}[c + d x]^2)^2} + \frac{5 \text{Coth}[c + d x]}{8 (a + b)^2 d (a + b - b \text{Tanh}[c + d x]^2)}$$

Result (type 3, 981 leaves):

$$\left((a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6 \right. \\ \left. - \left(\left(15 i b \text{ArcTan}[\text{Sech}[dx]] \left(-\frac{i \text{Cosh}[2c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4c] - b \text{Sinh}[4c]}} + \frac{i \text{Sinh}[2c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4c] - b \text{Sinh}[4c]}} \right) \right. \right. \right. \\ \left. \left. (-a \text{Sinh}[dx] - 2b \text{Sinh}[dx] + a \text{Sinh}[2c + dx]) \right) \text{Cosh}[2c] \right) / \left(64 \sqrt{a + b} d \sqrt{b \text{Cosh}[4c] - b \text{Sinh}[4c]} \right) \right) + \\ \left(15 i b \text{ArcTan}[\text{Sech}[dx]] \left(-\frac{i \text{Cosh}[2c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4c] - b \text{Sinh}[4c]}} + \frac{i \text{Sinh}[2c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4c] - b \text{Sinh}[4c]}} \right) \right. \\ \left. (-a \text{Sinh}[dx] - 2b \text{Sinh}[dx] + a \text{Sinh}[2c + dx]) \right) \text{Sinh}[2c] \Big/ \left(64 \sqrt{a + b} d \sqrt{b \text{Cosh}[4c] - b \text{Sinh}[4c]} \right) \Big) \Big/ \\ \left((a + b)^3 (a + b \text{Sech}[c + dx]^2)^3 \right) + \frac{1}{512 a^2 (a + b)^3 d (a + b \text{Sech}[c + dx]^2)^3} (a + 2b + a \text{Cosh}[2c + 2dx])$$

Csch[c]

Csch[c + dx]

Sech[2c]

Sech[c + dx]^6

$$\begin{aligned} & (-32 a^4 \text{Sinh}[dx] - 64 a^3 b \text{Sinh}[dx] + 22 a^2 b^2 \text{Sinh}[dx] + 80 a b^3 \text{Sinh}[dx] + 16 b^4 \text{Sinh}[dx] + 32 a^4 \text{Sinh}[3dx] + 46 a^3 b \text{Sinh}[3dx] - \\ & 54 a^2 b^2 \text{Sinh}[3dx] - 8 a b^3 \text{Sinh}[3dx] - 48 a^4 \text{Sinh}[2c - dx] - 128 a^3 b \text{Sinh}[2c - dx] - 106 a^2 b^2 \text{Sinh}[2c - dx] + \\ & 80 a b^3 \text{Sinh}[2c - dx] + 16 b^4 \text{Sinh}[2c - dx] + 48 a^4 \text{Sinh}[2c + dx] + 146 a^3 b \text{Sinh}[2c + dx] + 182 a^2 b^2 \text{Sinh}[2c + dx] + \\ & 80 a b^3 \text{Sinh}[2c + dx] + 16 b^4 \text{Sinh}[2c + dx] - 32 a^4 \text{Sinh}[4c + dx] - 82 a^3 b \text{Sinh}[4c + dx] - 54 a^2 b^2 \text{Sinh}[4c + dx] - \\ & 80 a b^3 \text{Sinh}[4c + dx] - 16 b^4 \text{Sinh}[4c + dx] - 8 a^4 \text{Sinh}[2c + 3dx] + 18 a^3 b \text{Sinh}[2c + 3dx] + 54 a^2 b^2 \text{Sinh}[2c + 3dx] + \\ & 8 a b^3 \text{Sinh}[2c + 3dx] + 32 a^4 \text{Sinh}[4c + 3dx] + 73 a^3 b \text{Sinh}[4c + 3dx] + 24 a^2 b^2 \text{Sinh}[4c + 3dx] + 8 a b^3 \text{Sinh}[4c + 3dx] - \\ & 8 a^4 \text{Sinh}[6c + 3dx] - 9 a^3 b \text{Sinh}[6c + 3dx] - 24 a^2 b^2 \text{Sinh}[6c + 3dx] - 8 a b^3 \text{Sinh}[6c + 3dx] + 8 a^4 \text{Sinh}[2c + 5dx] - \\ & 9 a^3 b \text{Sinh}[2c + 5dx] - 2 a^2 b^2 \text{Sinh}[2c + 5dx] + 9 a^3 b \text{Sinh}[4c + 5dx] + 2 a^2 b^2 \text{Sinh}[4c + 5dx] + 8 a^4 \text{Sinh}[6c + 5dx]) \end{aligned}$$

Problem 47: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^3}{(a + b \text{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 213 leaves, 7 steps):

$$\frac{\sqrt{b} (15 a^2 - 10 a b - b^2) \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c + d x]}{\sqrt{b}}\right] + (a - 5 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{8 a^{3/2} (a + b)^4 d} + \frac{(2 a - b) b \text{Cosh}[c + d x]}{4 a (a + b)^2 d (b + a \text{Cosh}[c + d x]^2)^2} - \frac{(4 a^2 - 9 a b - b^2) \text{Cosh}[c + d x]}{8 a (a + b)^3 d (b + a \text{Cosh}[c + d x]^2)} - \frac{\text{Cosh}[c + d x] \text{Coth}[c + d x]^2}{2 (a + b) d (b + a \text{Cosh}[c + d x]^2)^2}$$

Result (type 3, 524 leaves):

$$\frac{1}{64 (a + b)^4 d (a + b \text{Sech}[c + d x]^2)^3} \left((a + 2 b + a \text{Cosh}[2 (c + d x)]) \text{Sech}[c + d x]^5 \left(-\frac{8 b^2 (a + b)^2}{a} + \frac{2 b (a + b) (9 a + b) (a + 2 b + a \text{Cosh}[2 (c + d x)])}{a} + \frac{1}{a^{3/2}} \sqrt{b} (-15 a^2 + 10 a b + b^2) \right. \right. \\ \left. \left. \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a + b}) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} - i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) \right. \\ \left. (a + 2 b + a \text{Cosh}[2 (c + d x)])^2 \text{Sech}[c + d x] + \frac{1}{a^{3/2}} \sqrt{b} (-15 a^2 + 10 a b + b^2) \right. \\ \left. \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a + b}) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} + i \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) \right. \\ \left. (a + 2 b + a \text{Cosh}[2 (c + d x)])^2 \text{Sech}[c + d x] - (a + b) (a + 2 b + a \text{Cosh}[2 (c + d x)])^2 \text{Csch}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sech}[c + d x] + \right. \\ \left. 4 (a - 5 b) (a + 2 b + a \text{Cosh}[2 (c + d x)])^2 \text{Log}\left[\text{Cosh}\left[\frac{1}{2} (c + d x)\right]\right] \text{Sech}[c + d x] - 4 (a - 5 b) (a + 2 b + a \text{Cosh}[2 (c + d x)])^2 \right. \\ \left. \text{Log}\left[\text{Sinh}\left[\frac{1}{2} (c + d x)\right]\right] \text{Sech}[c + d x] - (a + b) (a + 2 b + a \text{Cosh}[2 (c + d x)])^2 \text{Sech}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sech}[c + d x] \right)$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^4}{(a + b \text{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$-\frac{5(3a-4b)\sqrt{b}\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8(a+b)^{9/2}d} + \frac{(a-2b)\operatorname{Coth}[c+dx]}{(a+b)^4d} -$$

$$\frac{\operatorname{Coth}[c+dx]^3}{3(a+b)^3d} - \frac{ab\operatorname{Tanh}[c+dx]}{4(a+b)^3d(a+b-b\operatorname{Tanh}[c+dx])^2} - \frac{(7a-4b)b\operatorname{Tanh}[c+dx]}{8(a+b)^4d(a+b-b\operatorname{Tanh}[c+dx])^2}$$

Result (type 3, 1228 leaves):

$$\left((3a-4b)(a+2b+a\operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \right.$$

$$\left(\left(5i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a\operatorname{Sinh}[dx] - 2b\operatorname{Sinh}[dx] + a\operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \left(64\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]} \right) -$$

$$\left(5i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} \right) \right.$$

$$\left. \left. (-a\operatorname{Sinh}[dx] - 2b\operatorname{Sinh}[dx] + a\operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / \left(64\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]} \right) \left. \right) /$$

$$\left((a+b)^4(a+b\operatorname{Sech}[c+dx]^2)^3 \right) + \frac{1}{6144a(a+b)^4d(a+b\operatorname{Sech}[c+dx]^2)^3} (a+2b+a\operatorname{Cosh}[2c+2dx])$$

$\operatorname{Csch}[c]$

$\operatorname{Csch}[c+dx]^3$

$\operatorname{Sech}[2c]$

$\operatorname{Sech}[c+dx]^6$

$$\begin{aligned} & (-176a^4\operatorname{Sinh}[dx] - 488a^3b\operatorname{Sinh}[dx] - 252a^2b^2\operatorname{Sinh}[dx] - 504ab^3\operatorname{Sinh}[dx] - 144b^4\operatorname{Sinh}[dx] + 96a^4\operatorname{Sinh}[3dx] + 71a^3b\operatorname{Sinh}[3dx] - \\ & 344a^2b^2\operatorname{Sinh}[3dx] + 1208a^3b\operatorname{Sinh}[3dx] - 48b^4\operatorname{Sinh}[3dx] - 224a^4\operatorname{Sinh}[2c-dx] - 576a^3b\operatorname{Sinh}[2c-dx] - 124a^2b^2\operatorname{Sinh}[2c-dx] + \\ & 2184a^3b\operatorname{Sinh}[2c-dx] - 144b^4\operatorname{Sinh}[2c-dx] + 224a^4\operatorname{Sinh}[2c+dx] + 657a^3b\operatorname{Sinh}[2c+dx] + 538a^2b^2\operatorname{Sinh}[2c+dx] - \\ & 984a^3b\operatorname{Sinh}[2c+dx] - 144b^4\operatorname{Sinh}[2c+dx] - 176a^4\operatorname{Sinh}[4c+dx] - 569a^3b\operatorname{Sinh}[4c+dx] - 666a^2b^2\operatorname{Sinh}[4c+dx] - \\ & 1704a^3b\operatorname{Sinh}[4c+dx] + 144b^4\operatorname{Sinh}[4c+dx] - 48a^4\operatorname{Sinh}[2c+3dx] - 111a^3b\operatorname{Sinh}[2c+3dx] - 360a^2b^2\operatorname{Sinh}[2c+3dx] - \\ & 312a^3b\operatorname{Sinh}[2c+3dx] + 48b^4\operatorname{Sinh}[2c+3dx] + 96a^4\operatorname{Sinh}[4c+3dx] + 152a^3b\operatorname{Sinh}[4c+3dx] - 146a^2b^2\operatorname{Sinh}[4c+3dx] + \\ & 728a^3b\operatorname{Sinh}[4c+3dx] + 48b^4\operatorname{Sinh}[4c+3dx] - 48a^4\operatorname{Sinh}[6c+3dx] - 192a^3b\operatorname{Sinh}[6c+3dx] - 558a^2b^2\operatorname{Sinh}[6c+3dx] + \\ & 168a^3b\operatorname{Sinh}[6c+3dx] - 48b^4\operatorname{Sinh}[6c+3dx] - 16a^4\operatorname{Sinh}[2c+5dx] + 598a^2b^2\operatorname{Sinh}[2c+5dx] - 48ab^3\operatorname{Sinh}[2c+5dx] - \\ & 72a^3b\operatorname{Sinh}[4c+5dx] - 150a^2b^2\operatorname{Sinh}[4c+5dx] + 48ab^3\operatorname{Sinh}[4c+5dx] - 16a^4\operatorname{Sinh}[6c+5dx] - 27a^3b\operatorname{Sinh}[6c+5dx] + \\ & 388a^2b^2\operatorname{Sinh}[6c+5dx] - 45a^3b\operatorname{Sinh}[8c+5dx] + 60a^2b^2\operatorname{Sinh}[8c+5dx] - 16a^4\operatorname{Sinh}[4c+7dx] + 83a^3b\operatorname{Sinh}[4c+7dx] - \\ & 6a^2b^2\operatorname{Sinh}[4c+7dx] - 27a^3b\operatorname{Sinh}[6c+7dx] + 6a^2b^2\operatorname{Sinh}[6c+7dx] - 16a^4\operatorname{Sinh}[8c+7dx] + 56a^3b\operatorname{Sinh}[8c+7dx]) \end{aligned}$$

Problem 62: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + d x]^2 (a + b \operatorname{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{(a + b)^2 \operatorname{Tanh}[c + d x]}{d} - \frac{2 b (a + b) \operatorname{Tanh}[c + d x]^3}{3 d} + \frac{b^2 \operatorname{Tanh}[c + d x]^5}{5 d}$$

Result (type 3, 116 leaves):

$$\frac{a^2 \operatorname{Tanh}[c + d x]}{d} + \frac{4 a b \operatorname{Tanh}[c + d x]}{3 d} + \frac{8 b^2 \operatorname{Tanh}[c + d x]}{15 d} + \frac{2 a b \operatorname{Sech}[c + d x]^2 \operatorname{Tanh}[c + d x]}{3 d} + \frac{4 b^2 \operatorname{Sech}[c + d x]^2 \operatorname{Tanh}[c + d x]}{15 d} + \frac{b^2 \operatorname{Sech}[c + d x]^4 \operatorname{Tanh}[c + d x]}{5 d}$$

Problem 64: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + d x]^4 (a + b \operatorname{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$\frac{(a + b)^2 \operatorname{Tanh}[c + d x]}{d} - \frac{(a + b) (a + 3 b) \operatorname{Tanh}[c + d x]^3}{3 d} + \frac{b (2 a + 3 b) \operatorname{Tanh}[c + d x]^5}{5 d} - \frac{b^2 \operatorname{Tanh}[c + d x]^7}{7 d}$$

Result (type 3, 190 leaves):

$$\frac{2 a^2 \operatorname{Tanh}[c + d x]}{3 d} + \frac{16 a b \operatorname{Tanh}[c + d x]}{15 d} + \frac{16 b^2 \operatorname{Tanh}[c + d x]}{35 d} + \frac{a^2 \operatorname{Sech}[c + d x]^2 \operatorname{Tanh}[c + d x]}{3 d} + \frac{8 a b \operatorname{Sech}[c + d x]^2 \operatorname{Tanh}[c + d x]}{15 d} + \frac{8 b^2 \operatorname{Sech}[c + d x]^2 \operatorname{Tanh}[c + d x]}{35 d} + \frac{2 a b \operatorname{Sech}[c + d x]^4 \operatorname{Tanh}[c + d x]}{5 d} + \frac{6 b^2 \operatorname{Sech}[c + d x]^4 \operatorname{Tanh}[c + d x]}{35 d} + \frac{b^2 \operatorname{Sech}[c + d x]^6 \operatorname{Tanh}[c + d x]}{7 d}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cosh}[c + d x] (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{3 b (8 a^2 + 4 a b + b^2) \operatorname{ArcTan}[\operatorname{Sinh}[c + d x]]}{8 d} + \frac{a^3 \operatorname{Sinh}[c + d x]}{d} + \frac{3 b^2 (4 a + b) \operatorname{Sech}[c + d x] \operatorname{Tanh}[c + d x]}{8 d} + \frac{b^3 \operatorname{Sech}[c + d x]^3 \operatorname{Tanh}[c + d x]}{4 d}$$

Result (type 3, 189 leaves):

$$\frac{1}{d (a + 2b + a \operatorname{Cosh}[2(c + dx)])^3} (b + a \operatorname{Cosh}[c + dx]^2)^3 \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^4$$

$$\left(6b (8a^2 + 4ab + b^2) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Cosh}[c] \operatorname{Cosh}[c + dx]^4 + 2b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c] + 3b^2 (4a + b) \operatorname{Cosh}[c + dx]^3 \operatorname{Sinh}[c] + 4a^3 \operatorname{Cosh}[dx] \operatorname{Cosh}[c + dx]^4 \operatorname{Sinh}[2c] + 2b^3 \operatorname{Sinh}[dx] + 3b^2 (4a + b) \operatorname{Cosh}[c + dx]^2 \operatorname{Sinh}[dx] + 8a^3 \operatorname{Cosh}[c]^2 \operatorname{Cosh}[c + dx]^4 \operatorname{Sinh}[dx] \right)$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + dx]^2 (a + b \operatorname{Sech}[c + dx]^2)^3 dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$\frac{(a + b)^3 \operatorname{Tanh}[c + dx]}{d} - \frac{b (a + b)^2 \operatorname{Tanh}[c + dx]^3}{d} + \frac{3b^2 (a + b) \operatorname{Tanh}[c + dx]^5}{5d} - \frac{b^3 \operatorname{Tanh}[c + dx]^7}{7d}$$

Result (type 3, 319 leaves):

$$\frac{1}{280d (a + 2b + a \operatorname{Cosh}[2(c + dx)])^3}$$

$$\operatorname{Sech}[c] \operatorname{Sech}[c + dx] (a + b \operatorname{Sech}[c + dx]^2)^3 (140 (5a^3 + 11a^2b + 10ab^2 + 4b^3) \operatorname{Sinh}[dx] - 35a (15a^2 + 26ab + 16b^2) \operatorname{Sinh}[2c + dx] + 525a^3 \operatorname{Sinh}[2c + 3dx] + 1260a^2b \operatorname{Sinh}[2c + 3dx] + 1176ab^2 \operatorname{Sinh}[2c + 3dx] + 336b^3 \operatorname{Sinh}[2c + 3dx] - 210a^3 \operatorname{Sinh}[4c + 3dx] - 210a^2b \operatorname{Sinh}[4c + 3dx] + 210a^3 \operatorname{Sinh}[4c + 5dx] + 490a^2b \operatorname{Sinh}[4c + 5dx] + 392ab^2 \operatorname{Sinh}[4c + 5dx] + 112b^3 \operatorname{Sinh}[4c + 5dx] - 35a^3 \operatorname{Sinh}[6c + 5dx] + 35a^3 \operatorname{Sinh}[6c + 7dx] + 70a^2b \operatorname{Sinh}[6c + 7dx] + 56ab^2 \operatorname{Sinh}[6c + 7dx] + 16b^3 \operatorname{Sinh}[6c + 7dx])$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + dx]^3 (a + b \operatorname{Sech}[c + dx]^2)^3 dx$$

Optimal (type 3, 196 leaves, 6 steps):

$$\frac{(64a^3 + 144a^2b + 120ab^2 + 35b^3) \operatorname{ArcTan}[\operatorname{Sinh}[c + dx]]}{128d} +$$

$$\frac{(64a^3 + 144a^2b + 120ab^2 + 35b^3) \operatorname{Sech}[c + dx] \operatorname{Tanh}[c + dx]}{128d} + \frac{b (72a^2 + 92ab + 35b^2) \operatorname{Sech}[c + dx]^3 \operatorname{Tanh}[c + dx]}{192d} +$$

$$\frac{b (12a + 7b) \operatorname{Sech}[c + dx]^5 (a + b + a \operatorname{Sinh}[c + dx]^2) \operatorname{Tanh}[c + dx]}{48d} + \frac{b \operatorname{Sech}[c + dx]^7 (a + b + a \operatorname{Sinh}[c + dx]^2)^2 \operatorname{Tanh}[c + dx]}{8d}$$

Result (type 3, 629 leaves):

$$\begin{aligned}
& \frac{(64 a^3 + 144 a^2 b + 120 a b^2 + 35 b^3) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Cosh}[c + dx]^6 (a + b \operatorname{Sech}[c + dx]^2)^3}{8 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3} + \\
& \frac{\operatorname{Cosh}[c + dx] \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + dx]^2)^3 (24 a b^2 \operatorname{Sinh}[c] + 7 b^3 \operatorname{Sinh}[c])}{6 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3} + \\
& \frac{\operatorname{Cosh}[c + dx]^3 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + dx]^2)^3 (144 a^2 b \operatorname{Sinh}[c] + 120 a b^2 \operatorname{Sinh}[c] + 35 b^3 \operatorname{Sinh}[c])}{24 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3} + \\
& \left(\operatorname{Cosh}[c + dx]^5 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + dx]^2)^3 (64 a^3 \operatorname{Sinh}[c] + 144 a^2 b \operatorname{Sinh}[c] + 120 a b^2 \operatorname{Sinh}[c] + 35 b^3 \operatorname{Sinh}[c]) \right) / \\
& \left(16 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3 \right) + \frac{b^3 \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^2 (a + b \operatorname{Sech}[c + dx]^2)^3 \operatorname{Sinh}[dx]}{d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3} + \\
& \frac{\operatorname{Sech}[c] (a + b \operatorname{Sech}[c + dx]^2)^3 (24 a b^2 \operatorname{Sinh}[dx] + 7 b^3 \operatorname{Sinh}[dx])}{6 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3} + \\
& \frac{\operatorname{Cosh}[c + dx]^2 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + dx]^2)^3 (144 a^2 b \operatorname{Sinh}[dx] + 120 a b^2 \operatorname{Sinh}[dx] + 35 b^3 \operatorname{Sinh}[dx])}{24 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3} + \\
& \left(\operatorname{Cosh}[c + dx]^4 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + dx]^2)^3 (64 a^3 \operatorname{Sinh}[dx] + 144 a^2 b \operatorname{Sinh}[dx] + 120 a b^2 \operatorname{Sinh}[dx] + 35 b^3 \operatorname{Sinh}[dx]) \right) / \\
& \left(16 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3 \right) + \frac{b^3 \operatorname{Sech}[c + dx] (a + b \operatorname{Sech}[c + dx]^2)^3 \operatorname{Tanh}[c]}{d (a + 2 b + a \operatorname{Cosh}[2 c + 2 dx])^3}
\end{aligned}$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + dx]^4 (a + b \operatorname{Sech}[c + dx]^2)^3 dx$$

Optimal (type 3, 108 leaves, 3 steps):

$$\frac{(a + b)^3 \operatorname{Tanh}[c + dx]}{d} - \frac{(a + b)^2 (a + 4 b) \operatorname{Tanh}[c + dx]^3}{3 d} + \frac{3 b (a + b) (a + 2 b) \operatorname{Tanh}[c + dx]^5}{5 d} - \frac{b^2 (3 a + 4 b) \operatorname{Tanh}[c + dx]^7}{7 d} + \frac{b^3 \operatorname{Tanh}[c + dx]^9}{9 d}$$

Result (type 3, 348 leaves):

$$\begin{aligned}
& \frac{1}{40320 d} \\
& \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^9 (63 (125 a^3 + 324 a^2 b + 312 a b^2 + 128 b^3) \operatorname{Sinh}[dx] - 315 a (17 a^2 + 36 a b + 24 b^2) \operatorname{Sinh}[2 c + dx] + 6825 a^3 \operatorname{Sinh}[2 c + 3 dx] + \\
& 18648 a^2 b \operatorname{Sinh}[2 c + 3 dx] + 18144 a b^2 \operatorname{Sinh}[2 c + 3 dx] + 5376 b^3 \operatorname{Sinh}[2 c + 3 dx] - 1995 a^3 \operatorname{Sinh}[4 c + 3 dx] - \\
& 2520 a^2 b \operatorname{Sinh}[4 c + 3 dx] + 3465 a^3 \operatorname{Sinh}[4 c + 5 dx] + 9072 a^2 b \operatorname{Sinh}[4 c + 5 dx] + 7776 a b^2 \operatorname{Sinh}[4 c + 5 dx] + \\
& 2304 b^3 \operatorname{Sinh}[4 c + 5 dx] - 315 a^3 \operatorname{Sinh}[6 c + 5 dx] + 945 a^3 \operatorname{Sinh}[6 c + 7 dx] + 2268 a^2 b \operatorname{Sinh}[6 c + 7 dx] + 1944 a b^2 \operatorname{Sinh}[6 c + 7 dx] + \\
& 576 b^3 \operatorname{Sinh}[6 c + 7 dx] + 105 a^3 \operatorname{Sinh}[8 c + 9 dx] + 252 a^2 b \operatorname{Sinh}[8 c + 9 dx] + 216 a b^2 \operatorname{Sinh}[8 c + 9 dx] + 64 b^3 \operatorname{Sinh}[8 c + 9 dx])
\end{aligned}$$

Problem 76: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{b \text{ArcTan}\left[\frac{\sqrt{a} \text{Sinh}[c + d x]}{\sqrt{a + b}}\right]}{a^{3/2} \sqrt{a + b} d} + \frac{\text{Sinh}[c + d x]}{a d}$$

Result (type 3, 147 leaves):

$$\left(b \text{ArcTan}\left[\frac{\sqrt{a + b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} (\text{Cosh}[c] + \text{Sinh}[c])}}{\sqrt{a}}\right] \text{Cosh}[c] - \right. \\ \left. b \text{ArcTan}\left[\frac{\sqrt{a + b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} (\text{Cosh}[c] + \text{Sinh}[c])}}{\sqrt{a}}\right] \text{Sinh}[c] + \right. \\ \left. \sqrt{a} \sqrt{a + b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Sinh}[c + d x] \right) / \left(a^{3/2} \sqrt{a + b} d \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right)$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sech}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Sinh}[c + d x]}{\sqrt{a + b}}\right]}{\sqrt{a} \sqrt{a + b} d}$$

Result (type 3, 114 leaves):

$$\left(\text{ArcTan}\left[\frac{\sqrt{a + b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} (\text{Cosh}[c] + \text{Sinh}[c])}}{\sqrt{a}}\right] (a + 2 b + a \text{Cosh}[2(c + d x)]) \text{Sech}[c + d x]^2 (-\text{Cosh}[c] + \text{Sinh}[c]) \right) / \\ \left(2 \sqrt{a} \sqrt{a + b} d (a + b \text{Sech}[c + d x]^2) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right)$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sech}[c + d x]^3}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\text{ArcTan}[\text{Sinh}[c + d x]]}{b d} - \frac{\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \text{Sinh}[c + d x]}{\sqrt{a + b}}\right]}{b \sqrt{a + b} d}$$

Result (type 3, 194 leaves):

$$\left((a + 2 b + a \text{Cosh}[2(c + d x)]) \text{Sech}[c + d x]^2 \right. \\ \left. \left(\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a + b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} (\text{Cosh}[c] + \text{Sinh}[c])}}{\sqrt{a}}\right] \text{Cosh}[c] + 2 \sqrt{a + b} \text{ArcTan}\left[\text{Tanh}\left[\frac{1}{2}(c + d x)\right]\right] \right) \right. \\ \left. \left. \left(\sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} - \sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a + b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} (\text{Cosh}[c] + \text{Sinh}[c])}}{\sqrt{a}}\right] \text{Sinh}[c] \right) \right) \right) / \\ (2 b \sqrt{a + b} d (a + b \text{Sech}[c + d x]^2) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2})$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sech}[c + d x]^4}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$- \frac{a \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{b^{3/2} \sqrt{a + b} d} + \frac{\text{Tanh}[c + d x]}{b d}$$

Result (type 3, 182 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^2 \right. \\ \left. a \operatorname{ArcTan}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] (-\operatorname{Cosh}[2c] + \operatorname{Sinh}[2c]) + \right. \\ \left. \sqrt{a+b} \operatorname{Sech}[c] \operatorname{Sech}[c + dx] \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \operatorname{Sinh}[dx] \right) \Bigg) / \left(2b\sqrt{a+b} d (a + b \operatorname{Sech}[c + dx]^2) \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right)$$

Problem 81: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + dx]^5}{a + b \operatorname{Sech}[c + dx]^2} dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$-\frac{(2a - b) \operatorname{ArcTan}[\operatorname{Sinh}[c + dx]]}{2b^2 d} + \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c + dx]}{\sqrt{a+b}}\right]}{b^2 \sqrt{a+b} d} + \frac{\operatorname{Sech}[c + dx] \operatorname{Tanh}[c + dx]}{2bd}$$

Result (type 3, 213 leaves):

$$\frac{1}{4b^2 \sqrt{a+b} d (a + b \operatorname{Sech}[c + dx]^2) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}} \\ \operatorname{Cosh}[c] (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^2 \left(b\sqrt{a+b} \operatorname{Sech}[c]^2 \operatorname{Sech}[c + dx]^2 \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Sinh}[dx] + \right. \\ \left. 2a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \operatorname{Csch}[c + dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])}{\sqrt{a}} \right] (-1 + \operatorname{Tanh}[c]) - \right. \\ \left. \sqrt{a+b} \operatorname{Sech}[c] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \left(2(2a - b) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]\right] - b \operatorname{Sech}[c + dx] \operatorname{Tanh}[c] \right) \right)$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + dx]^6}{a + b \operatorname{Sech}[c + dx]^2} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{b^{5/2} \sqrt{a+b} d} - \frac{(a-b) \operatorname{Tanh}[c+dx]}{b^2 d} - \frac{\operatorname{Tanh}[c+dx]^3}{3 b d}$$

Result (type 3, 214 leaves):

$$\left((a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^2 \right. \\ \left. \left(3 a^2 \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a+2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c+dx])}{2 \sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}\right]} (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) + \right. \right. \\ \left. \left. \sqrt{a+b} \operatorname{Sech}[c+dx] \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 (\operatorname{Sech}[c] (-3a+2b+b \operatorname{Sech}[c+dx]^2) \operatorname{Sinh}[dx] + b \operatorname{Sech}[c+dx] \operatorname{Tanh}[c]) \right) \right) / \\ \left(6 b^2 \sqrt{a+b} d (a+b \operatorname{Sech}[c+dx]^2) \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right)$$

Problem 85: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cosh}[c+dx]}{(a+b \operatorname{Sech}[c+dx]^2)^2} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{b(4a+3b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c+dx]}{\sqrt{a+b}}\right]}{2 a^{5/2} (a+b)^{3/2} d} + \frac{\operatorname{Sinh}[c+dx]}{a^2 d} + \frac{b^2 \operatorname{Sinh}[c+dx]}{2 a^2 (a+b) d (a+b+a \operatorname{Sinh}[c+dx]^2)}$$

Result (type 3, 234 leaves):

$$\frac{1}{8 a^{5/2} d (a+b \operatorname{Sech}[c+dx]^2)^2} (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^3 \\ \left(\left(b(4a+3b) \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \operatorname{Csch}[c+dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])}}{\sqrt{a}}\right]} (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] \right. \right. \\ \left. \left. (\operatorname{Cosh}[c] - \operatorname{Sinh}[c]) \right) / \left((a+b)^{3/2} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} + 2 \sqrt{a} \operatorname{Cosh}[dx] (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] \operatorname{Sinh}[c] + \right. \right. \\ \left. \left. 2 \sqrt{a} \operatorname{Cosh}[c] (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] \operatorname{Sinh}[dx] + \frac{2 \sqrt{a} b^2 \operatorname{Tanh}[c+dx]}{a+b} \right) \right)$$

Problem 87: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^2}{(a + b \operatorname{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{2 \sqrt{b} (a + b)^{3/2} d} + \frac{\operatorname{Tanh}[c + d x]}{2 (a + b) d (a + b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 187 leaves):

$$\left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^4 \right. \\ \left. \left(\left(\operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])}{2 \sqrt{a + b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] (a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \right) \right. \right. \\ \left. \left. (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) \right) \right) / \left(\sqrt{a + b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right) + \\ \left. \operatorname{Sech}[2 c] \operatorname{Sinh}[2 d x] - \frac{(a + 2 b) \operatorname{Tanh}[2 c]}{a} \right) / \left(8 (a + b) d (a + b \operatorname{Sech}[c + d x]^2)^2 \right)$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^3}{(a + b \operatorname{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c + d x]}{\sqrt{a + b}}\right]}{2 \sqrt{a} (a + b)^{3/2} d} + \frac{\operatorname{Sinh}[c + d x]}{2 (a + b) d (a + b + a \operatorname{Sinh}[c + d x]^2)}$$

Result (type 3, 150 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^3 \right. \\ \left. \left(\left(\operatorname{ArcTan} \left[\frac{\sqrt{a+b} \operatorname{Csch}[c + dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])}}{\sqrt{a}} \right] (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx] \right. \right. \right. \\ \left. \left. \left. (-\operatorname{Cosh}[c] + \operatorname{Sinh}[c]) \right) \right) / \left(\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} + 2 \operatorname{Tanh}[c + dx] \right) \right) \right) / \left(8(a+b)d(a+b \operatorname{Sech}[c + dx]^2)^2 \right)$$

Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + dx]^5}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 101 leaves, 5 steps):

$$\frac{\operatorname{ArcTan}[\operatorname{Sinh}[c + dx]]}{b^2 d} - \frac{\sqrt{a} (2a + 3b) \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Sinh}[c + dx]}{\sqrt{a+b}} \right]}{2b^2 (a+b)^{3/2} d} - \frac{a \operatorname{Sinh}[c + dx]}{2b(a+b)d(a+b + a \operatorname{Sinh}[c + dx]^2)}$$

Result (type 3, 282 leaves):

$$\frac{1}{8b^2(a+b)^{3/2}d(a+b \operatorname{Sech}[c + dx]^2)^2 \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}} (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^3 \\ \left(\sqrt{a} (2a + 3b) \operatorname{ArcTan} \left[\frac{\sqrt{a+b} \operatorname{Csch}[c + dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])}}{\sqrt{a}} \right] \operatorname{Cosh}[c] (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx] - \right. \\ \left. (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx] \left(-4(a+b)^{3/2} \operatorname{ArcTan} \left[\operatorname{Tanh} \left[\frac{1}{2}(c + dx) \right] \right] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} + \sqrt{a} (2a + 3b) \operatorname{ArcTan} \left[\right. \right. \right. \\ \left. \left. \left. \frac{\sqrt{a+b} \operatorname{Csch}[c + dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])}}{\sqrt{a}} \right] \operatorname{Sinh}[c] \right) - 2ab\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}[c + dx] \right) \right)$$

Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + dx]^6}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 101 leaves, 5 steps):

$$-\frac{a(3a+4b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{2b^{5/2}(a+b)^{3/2}d} + \frac{\operatorname{Tanh}[c+dx]}{b^2d} + \frac{a^2\operatorname{Tanh}[c+dx]}{2b^2(a+b)d(a+b-b\operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 483 leaves):

$$\begin{aligned} & \left((3a+4b)(a+2b+a\operatorname{Cosh}[2c+2dx])^2\operatorname{Sech}[c+dx]^4 \right. \\ & \left(\left(\operatorname{ArcTan}\left[\operatorname{Sech}[dx]\left(-\frac{i\operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i\operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}}\right)\right] \right. \right. \\ & \left. \left. (-a\operatorname{Sinh}[dx]-2b\operatorname{Sinh}[dx]+a\operatorname{Sinh}[2c+dx])\operatorname{Cosh}[2c] \right) / \left(8b^2\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]} \right) - \right. \\ & \left. \left(\operatorname{ArcTan}\left[\operatorname{Sech}[dx]\left(-\frac{i\operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i\operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}}\right)\right] \right. \right. \\ & \left. \left. (-a\operatorname{Sinh}[dx]-2b\operatorname{Sinh}[dx]+a\operatorname{Sinh}[2c+dx])\operatorname{Sinh}[2c] \right) / \left(8b^2\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]} \right) \right) \Bigg) / \\ & \left((a+b)(a+b\operatorname{Sech}[c+dx]^2)^2 \right) + \frac{(a+2b+a\operatorname{Cosh}[2c+2dx])^2\operatorname{Sech}[c]\operatorname{Sech}[c+dx]^5\operatorname{Sinh}[dx]}{4b^2d(a+b\operatorname{Sech}[c+dx]^2)^2} + \\ & \frac{(a+2b+a\operatorname{Cosh}[2c+2dx])\operatorname{Sech}[2c]\operatorname{Sech}[c+dx]^4(-a^2\operatorname{Sinh}[2c]-2ab\operatorname{Sinh}[2c]+a^2\operatorname{Sinh}[2dx])}{8b^2(a+b)d(a+b\operatorname{Sech}[c+dx]^2)^2} \end{aligned}$$

Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+dx]^7}{(a+b\operatorname{Sech}[c+dx]^2)^2} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{(4a-b)\operatorname{ArcTan}[\operatorname{Sinh}[c+dx]]}{2b^3d} + \frac{a^{3/2}(4a+5b)\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Sinh}[c+dx]}{\sqrt{a+b}}\right]}{2b^3(a+b)^{3/2}d} + \frac{a(2a+b)\operatorname{Sinh}[c+dx]}{2b^2(a+b)d(a+b+a\operatorname{Sinh}[c+dx]^2)} + \frac{\operatorname{Sech}[c+dx]\operatorname{Tanh}[c+dx]}{2bd(a+b+a\operatorname{Sinh}[c+dx]^2)}$$

Result (type 3, 1144 leaves):

$$-\frac{(4a-b)\operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{c}{2} + \frac{dx}{2}\right]\right](a+2b+a\operatorname{Cosh}[2c+2dx])^2\operatorname{Sech}[c+dx]^4}{4b^3d(a+b\operatorname{Sech}[c+dx]^2)^2} +$$

$$\begin{aligned}
& \frac{\text{Cosh}\left[\frac{c}{2}\right] (a + 2b + a \text{Cosh}[2c + 2dx])^2 \text{Sech}[c] \text{Sech}[c + dx]^5 \text{Sinh}\left[\frac{c}{2}\right]}{4b^2 d (a + b \text{Sech}[c + dx]^2)^2} + \left((4a^3 + 5a^2b) (a + 2b + a \text{Cosh}[2c + 2dx])^2 \right. \\
& \left. \text{Sech}[c + dx]^4 \left(- \frac{\text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Cosh}[c]}{16\sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} + \right. \right. \\
& \left. \left. \frac{\text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Sinh}[c]}{16\sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) / \\
& \left((a + b) (a + b \text{Sech}[c + dx]^2)^2 \right) + \left((4a + 5b) (a + 2b + a \text{Cosh}[2c + 2dx])^2 \text{Sech}[c + dx]^4 \right. \\
& \left. \left(- \frac{a^{3/2} \text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Cosh}[c]}{16b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} + \right. \right. \\
& \left. \left. \frac{a^{3/2} \text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Sinh}[c]}{16b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) / \\
& \left((a + b) (a + b \text{Sech}[c + dx]^2)^2 \right) + \left((4a^3 + 5a^2b) (a + 2b + a \text{Cosh}[2c + 2dx])^2 \text{Sech}[c + dx]^4 \right. \\
& \left. \left(\frac{i \text{Cosh}[c] \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]]}{32\sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} - \frac{i \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]] \text{Sinh}[c]}{32\sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) / \\
& \left((a + b) (a + b \text{Sech}[c + dx]^2)^2 \right) + \left((4a + 5b) (a + 2b + a \text{Cosh}[2c + 2dx])^2 \text{Sech}[c + dx]^4 \right. \\
& \left. \left(- \frac{i a^{3/2} \text{Cosh}[c] \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]]}{32b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} + \frac{i a^{3/2} \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]] \text{Sinh}[c]}{32b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) /
\end{aligned}$$

$$\left((a+b) (a+b \operatorname{Sech}[c+dx]^2)^2 \right) + \frac{(a+2b+a \operatorname{Cosh}[2c+2dx])^2 \operatorname{Sech}[c] \operatorname{Sech}[c+dx]^6 \operatorname{Sinh}[dx]}{8b^2 d (a+b \operatorname{Sech}[c+dx]^2)^2} +$$

$$\frac{a^2 (a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[c+dx]^3 \operatorname{Tanh}[c+dx]}{4b^2 (a+b) d (a+b \operatorname{Sech}[c+dx]^2)^2}$$

Problem 96: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+dx]^2}{(a+b \operatorname{Sech}[c+dx]^2)^3} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8\sqrt{b} (a+b)^{5/2} d} + \frac{\operatorname{Tanh}[c+dx]}{4(a+b) d (a+b-b \operatorname{Tanh}[c+dx]^2)^2} + \frac{3 \operatorname{Tanh}[c+dx]}{8(a+b)^2 d (a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 258 leaves):

$$\left((a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^6 \right.$$

$$\left. \left(\left(3 \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a+2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c+dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] (a+2b+a \operatorname{Cosh}[2(c+dx)])^2 \right. \right.$$

$$\left. \left. (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) / \left(\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right) + \frac{4b(a+b) \operatorname{Sech}[2c] ((a+2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{a^2} \right.$$

$$\left. \left. \left. \frac{(a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[2c] ((5a^2+16ab+8b^2) \operatorname{Sinh}[2c] - a(5a+2b) \operatorname{Sinh}[2dx])}{a^2} \right) \right) / (64(a+b)^2 d (a+b \operatorname{Sech}[c+dx]^2)^3)$$

Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+dx]^4}{(a+b \operatorname{Sech}[c+dx]^2)^3} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$\frac{(a+4b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8b^{3/2}(a+b)^{5/2}d} - \frac{a \operatorname{Tanh}[c+dx]}{4b(a+b)d(a+b-b \operatorname{Tanh}[c+dx]^2)^2} + \frac{(a+4b) \operatorname{Tanh}[c+dx]}{8b(a+b)^2d(a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 507 leaves):

$$\begin{aligned} & \left((a+4b)(a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \right. \\ & \left. - \left(\left(\operatorname{ArcTan}\left[\operatorname{Sech}[dx]\right] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} \right) \right. \right. \right. \\ & \left. \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \left(64b\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]} \right) \right) + \\ & \left(\operatorname{ArcTan}\left[\operatorname{Sech}[dx]\right] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} \right) \right. \\ & \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / \left(64b\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]} \right) \right) / \\ & \left((a+b)^2(a+b \operatorname{Sech}[c+dx]^2)^3 \right) + \frac{(a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^6 (-a \operatorname{Sinh}[2c] - 2b \operatorname{Sinh}[2c] + a \operatorname{Sinh}[2dx])}{16a(a+b)d(a+b \operatorname{Sech}[c+dx]^2)^3} + \\ & \left((a+2b+a \operatorname{Cosh}[2c+2dx])^2 \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^6 \right. \\ & \left. (a \operatorname{Sinh}[2c] + 4b \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx] + 2b \operatorname{Sinh}[2dx]) \right) / \left(64b(a+b)^2d(a+b \operatorname{Sech}[c+dx]^2)^3 \right) \end{aligned}$$

Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+dx]^7}{(a+b \operatorname{Sech}[c+dx]^2)^3} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$\frac{\operatorname{ArcTan}[\operatorname{Sinh}[c+dx]]}{b^3d} - \frac{\sqrt{a}(8a^2+20ab+15b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c+dx]}{\sqrt{a+b}}\right]}{8b^3(a+b)^{5/2}d} - \frac{a \operatorname{Sinh}[c+dx]}{4b(a+b)d(a+b+a \operatorname{Sinh}[c+dx]^2)^2} - \frac{a(4a+7b) \operatorname{Sinh}[c+dx]}{8b^2(a+b)^2d(a+b+a \operatorname{Sinh}[c+dx]^2)}$$

Result (type 3, 1120 leaves):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\text{Tanh}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6}{4b^3 d (a + b \text{Sech}[c + dx]^2)^3} + \left((8a^3 + 20a^2b + 15ab^2) (a + 2b + a \text{Cosh}[2c + 2dx])^3 \right. \\
& \left. \text{Sech}[c + dx]^6 \frac{\left(\text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Cosh}[c] \right)}{128 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right. \\
& \left. \left. \frac{\text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Sinh}[c] \right)}{128 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) / \\
& \left((a+b)^2 (a + b \text{Sech}[c + dx]^2)^3 \right) + \left((8a^2 + 20ab + 15b^2) (a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6 \right. \\
& \left. \frac{\left(\sqrt{a} \text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Cosh}[c] \right)}{128 b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right. \\
& \left. \frac{\sqrt{a} \text{ArcTan}\left[\text{Csch}[c + dx] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right)\right] \text{Sinh}[c] \right)}{128 b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) / \\
& \left((a+b)^2 (a + b \text{Sech}[c + dx]^2)^3 \right) + \left((8a^3 + 20a^2b + 15ab^2) (a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6 \right. \\
& \left. \left(- \frac{i \text{Cosh}[c] \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]]}{256 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} + \frac{i \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]] \text{Sinh}[c]}{256 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) / \\
& \left((a+b)^2 (a + b \text{Sech}[c + dx]^2)^3 \right) + \left((8a^2 + 20ab + 15b^2) (a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6 \right. \\
& \left. \left(\frac{i \sqrt{a} \text{Cosh}[c] \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]]}{256 b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} - \frac{i \sqrt{a} \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]] \text{Sinh}[c]}{256 b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} \right) \right) / \left((a+b)^2 (a + b \text{Sech}[c + dx]^2)^3 \right) +
\end{aligned}$$

$$\frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^6 (-4a^2 \operatorname{Sinh}[c + dx] - 7ab \operatorname{Sinh}[c + dx])}{32b^2 (a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^3} -$$

$$\frac{a (a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Sech}[c + dx]^5 \operatorname{Tanh}[c + dx]}{8b (a + b) d (a + b \operatorname{Sech}[c + dx]^2)^3}$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + dx]^2)^2 \operatorname{Tanh}[c + dx]^4 dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$a^2 x - \frac{a^2 \operatorname{Tanh}[c + dx]}{d} - \frac{a^2 \operatorname{Tanh}[c + dx]^3}{3d} + \frac{b(2a + b) \operatorname{Tanh}[c + dx]^5}{5d} - \frac{b^2 \operatorname{Tanh}[c + dx]^7}{7d}$$

Result (type 3, 395 leaves):

$$\frac{1}{13440d} \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^7$$

$$(3675a^2 dx \operatorname{Cosh}[dx] + 3675a^2 dx \operatorname{Cosh}[2c + dx] + 2205a^2 dx \operatorname{Cosh}[2c + 3dx] + 2205a^2 dx \operatorname{Cosh}[4c + 3dx] + 735a^2 dx \operatorname{Cosh}[4c + 5dx] +$$

$$735a^2 dx \operatorname{Cosh}[6c + 5dx] + 105a^2 dx \operatorname{Cosh}[6c + 7dx] + 105a^2 dx \operatorname{Cosh}[8c + 7dx] - 5320a^2 \operatorname{Sinh}[dx] + 1680ab \operatorname{Sinh}[dx] +$$

$$840b^2 \operatorname{Sinh}[dx] + 4480a^2 \operatorname{Sinh}[2c + dx] - 1260ab \operatorname{Sinh}[2c + dx] + 420b^2 \operatorname{Sinh}[2c + dx] - 3780a^2 \operatorname{Sinh}[2c + 3dx] +$$

$$924ab \operatorname{Sinh}[2c + 3dx] - 168b^2 \operatorname{Sinh}[2c + 3dx] + 2100a^2 \operatorname{Sinh}[4c + 3dx] - 840ab \operatorname{Sinh}[4c + 3dx] -$$

$$420b^2 \operatorname{Sinh}[4c + 3dx] - 1540a^2 \operatorname{Sinh}[4c + 5dx] + 168ab \operatorname{Sinh}[4c + 5dx] + 84b^2 \operatorname{Sinh}[4c + 5dx] +$$

$$420a^2 \operatorname{Sinh}[6c + 5dx] - 420ab \operatorname{Sinh}[6c + 5dx] - 280a^2 \operatorname{Sinh}[6c + 7dx] + 84ab \operatorname{Sinh}[6c + 7dx] + 12b^2 \operatorname{Sinh}[6c + 7dx])$$

Problem 114: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + dx]^2)^2 \operatorname{Tanh}[c + dx]^2 dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$a^2 x - \frac{a^2 \operatorname{Tanh}[c + dx]}{d} + \frac{b(2a + b) \operatorname{Tanh}[c + dx]^3}{3d} - \frac{b^2 \operatorname{Tanh}[c + dx]^5}{5d}$$

Result (type 3, 281 leaves):

$$\frac{1}{480 d} \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^5$$

$$\left(150 a^2 d x \operatorname{Cosh}[d x] + 150 a^2 d x \operatorname{Cosh}[2 c + d x] + 75 a^2 d x \operatorname{Cosh}[2 c + 3 d x] + 75 a^2 d x \operatorname{Cosh}[4 c + 3 d x] + 15 a^2 d x \operatorname{Cosh}[4 c + 5 d x] + 15 a^2 d x \operatorname{Cosh}[6 c + 5 d x] - 180 a^2 \operatorname{Sinh}[d x] + 80 a b \operatorname{Sinh}[d x] - 20 b^2 \operatorname{Sinh}[d x] + 120 a^2 \operatorname{Sinh}[2 c + d x] - 120 a b \operatorname{Sinh}[2 c + d x] - 60 b^2 \operatorname{Sinh}[2 c + d x] - 120 a^2 \operatorname{Sinh}[2 c + 3 d x] + 40 a b \operatorname{Sinh}[2 c + 3 d x] + 20 b^2 \operatorname{Sinh}[2 c + 3 d x] + 30 a^2 \operatorname{Sinh}[4 c + 3 d x] - 60 a b \operatorname{Sinh}[4 c + 3 d x] - 30 a^2 \operatorname{Sinh}[4 c + 5 d x] + 20 a b \operatorname{Sinh}[4 c + 5 d x] + 4 b^2 \operatorname{Sinh}[4 c + 5 d x] \right)$$

Problem 116: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b(2a + b) \operatorname{Tanh}[c + d x]}{d} - \frac{b^2 \operatorname{Tanh}[c + d x]^3}{3d}$$

Result (type 3, 106 leaves):

$$\left(4 (b + a \operatorname{Cosh}[c + d x]^2)^2 \operatorname{Sech}[c + d x]^3 \right. \\ \left. (3 a^2 d x \operatorname{Cosh}[c + d x]^3 + b^2 \operatorname{Sech}[c] \operatorname{Sinh}[d x] + 2 b (3 a + b) \operatorname{Cosh}[c + d x]^2 \operatorname{Sech}[c] \operatorname{Sinh}[d x] + b^2 \operatorname{Cosh}[c + d x] \operatorname{Tanh}[c]) \right) / \left(3 d (a + 2 b + a \operatorname{Cosh}[2(c + d x)])^2 \right)$$

Problem 118: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^2 (a + b \operatorname{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$a^2 x - \frac{(a + b)^2 \operatorname{Coth}[c + d x]}{d} - \frac{b^2 \operatorname{Tanh}[c + d x]}{d}$$

Result (type 3, 82 leaves):

$$\left(4 (b + a \operatorname{Cosh}[c + d x]^2)^2 \operatorname{Sech}[c + d x] \left(a^2 d x \operatorname{Cosh}[c + d x] + \left((a + b)^2 \operatorname{Coth}[c + d x] \operatorname{Csch}[c] - b^2 \operatorname{Sech}[c] \right) \operatorname{Sinh}[d x] \right) \right) / \left(d (a + 2 b + a \operatorname{Cosh}[2(c + d x)])^2 \right)$$

Problem 120: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^4 (a + b \operatorname{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 46 leaves, 4 steps):

$$a^2 x - \frac{(a^2 - b^2) \operatorname{Coth}[c + d x]}{d} - \frac{(a + b)^2 \operatorname{Coth}[c + d x]^3}{3 d}$$

Result (type 3, 160 leaves):

$$\frac{1}{24 d} \operatorname{Csch}[c] \operatorname{Csch}[c + d x]^3 \\ (9 a^2 d x \operatorname{Cosh}[d x] - 9 a^2 d x \operatorname{Cosh}[2 c + d x] - 3 a^2 d x \operatorname{Cosh}[2 c + 3 d x] + 3 a^2 d x \operatorname{Cosh}[4 c + 3 d x] - 12 a^2 \operatorname{Sinh}[d x] + 12 b^2 \operatorname{Sinh}[d x] - \\ 12 a^2 \operatorname{Sinh}[2 c + d x] - 12 a b \operatorname{Sinh}[2 c + d x] + 8 a^2 \operatorname{Sinh}[2 c + 3 d x] + 4 a b \operatorname{Sinh}[2 c + 3 d x] - 4 b^2 \operatorname{Sinh}[2 c + 3 d x])$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^6 (a + b \operatorname{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 64 leaves, 4 steps):

$$a^2 x - \frac{a^2 \operatorname{Coth}[c + d x]}{d} - \frac{(a^2 - b^2) \operatorname{Coth}[c + d x]^3}{3 d} - \frac{(a + b)^2 \operatorname{Coth}[c + d x]^5}{5 d}$$

Result (type 3, 256 leaves):

$$\frac{1}{480 d} \operatorname{Csch}[c] \operatorname{Csch}[c + d x]^5 \\ (-150 a^2 d x \operatorname{Cosh}[d x] + 150 a^2 d x \operatorname{Cosh}[2 c + d x] + 75 a^2 d x \operatorname{Cosh}[2 c + 3 d x] - 75 a^2 d x \operatorname{Cosh}[4 c + 3 d x] - 15 a^2 d x \operatorname{Cosh}[4 c + 5 d x] + 15 a^2 d x \\ \operatorname{Cosh}[6 c + 5 d x] + 280 a^2 \operatorname{Sinh}[d x] + 120 a b \operatorname{Sinh}[d x] + 20 b^2 \operatorname{Sinh}[d x] + 180 a^2 \operatorname{Sinh}[2 c + d x] - 60 b^2 \operatorname{Sinh}[2 c + d x] - 140 a^2 \operatorname{Sinh}[2 c + 3 d x] + \\ 20 b^2 \operatorname{Sinh}[2 c + 3 d x] - 90 a^2 \operatorname{Sinh}[4 c + 3 d x] - 60 a b \operatorname{Sinh}[4 c + 3 d x] + 46 a^2 \operatorname{Sinh}[4 c + 5 d x] + 12 a b \operatorname{Sinh}[4 c + 5 d x] - 4 b^2 \operatorname{Sinh}[4 c + 5 d x])$$

Problem 124: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + d x]^2)^3 \operatorname{Tanh}[c + d x]^4 dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$a^3 x - \frac{a^3 \operatorname{Tanh}[c + d x]}{d} - \frac{a^3 \operatorname{Tanh}[c + d x]^3}{3 d} + \frac{b (3 a^2 + 3 a b + b^2) \operatorname{Tanh}[c + d x]^5}{5 d} - \frac{b^2 (3 a + 2 b) \operatorname{Tanh}[c + d x]^7}{7 d} + \frac{b^3 \operatorname{Tanh}[c + d x]^9}{9 d}$$

Result (type 3, 683 leaves):

$$\begin{aligned}
& \frac{8 a^3 x \operatorname{Cosh}[c+d x]^6 (a+b \operatorname{Sech}[c+d x]^2)^3}{(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \frac{8 \operatorname{Sech}[c](a+b \operatorname{Sech}[c+d x]^2)^3 (27 a b^2 \operatorname{Sinh}[c]-10 b^3 \operatorname{Sinh}[c])}{63 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \\
& \frac{8 \operatorname{Cosh}[c+d x]^2 \operatorname{Sech}[c](a+b \operatorname{Sech}[c+d x]^2)^3 (63 a^2 b \operatorname{Sinh}[c]-72 a b^2 \operatorname{Sinh}[c]+b^3 \operatorname{Sinh}[c])}{105 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \\
& \left(\frac{8 \operatorname{Cosh}[c+d x]^4 \operatorname{Sech}[c](a+b \operatorname{Sech}[c+d x]^2)^3 (105 a^3 \operatorname{Sinh}[c]-378 a^2 b \operatorname{Sinh}[c]+27 a b^2 \operatorname{Sinh}[c]+4 b^3 \operatorname{Sinh}[c])}{315 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \frac{8 b^3 \operatorname{Sech}[c] \operatorname{Sech}[c+d x]^3 (a+b \operatorname{Sech}[c+d x]^2)^3 \operatorname{Sinh}[d x]}{9 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \right. \\
& \left. \frac{8 \operatorname{Sech}[c] \operatorname{Sech}[c+d x](a+b \operatorname{Sech}[c+d x]^2)^3 (27 a b^2 \operatorname{Sinh}[d x]-10 b^3 \operatorname{Sinh}[d x])}{63 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} - \right. \\
& \left. \frac{8 \operatorname{Cosh}[c+d x]^5 \operatorname{Sech}[c](a+b \operatorname{Sech}[c+d x]^2)^3 (420 a^3 \operatorname{Sinh}[d x]-189 a^2 b \operatorname{Sinh}[d x]-54 a b^2 \operatorname{Sinh}[d x]-8 b^3 \operatorname{Sinh}[d x])}{315 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \frac{8 \operatorname{Cosh}[c+d x] \operatorname{Sech}[c](a+b \operatorname{Sech}[c+d x]^2)^3 (63 a^2 b \operatorname{Sinh}[d x]-72 a b^2 \operatorname{Sinh}[d x]+b^3 \operatorname{Sinh}[d x])}{105 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \right. \\
& \left. \frac{8 \operatorname{Cosh}[c+d x]^3 \operatorname{Sech}[c](a+b \operatorname{Sech}[c+d x]^2)^3 (105 a^3 \operatorname{Sinh}[d x]-378 a^2 b \operatorname{Sinh}[d x]+27 a b^2 \operatorname{Sinh}[d x]+4 b^3 \operatorname{Sinh}[d x])}{315 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \frac{8 b^3 \operatorname{Sech}[c+d x]^2 (a+b \operatorname{Sech}[c+d x]^2)^3 \operatorname{Tanh}[c]}{9 d(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} \right) /
\end{aligned}$$

Problem 126: Result more than twice size of optimal antiderivative.

$$\int (a+b \operatorname{Sech}[c+d x]^2)^3 \operatorname{Tanh}[c+d x]^2 dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$a^3 x - \frac{a^3 \operatorname{Tanh}[c+d x]}{d} + \frac{b(3 a^2+3 a b+b^2) \operatorname{Tanh}[c+d x]^3}{3 d} - \frac{b^2(3 a+2 b) \operatorname{Tanh}[c+d x]^5}{5 d} + \frac{b^3 \operatorname{Tanh}[c+d x]^7}{7 d}$$

Result (type 3, 479 leaves):

$$\begin{aligned}
& \frac{1}{13440 d} \operatorname{Sech}[c] \operatorname{Sech}[c+d x]^7 \\
& (3675 a^3 d x \operatorname{Cosh}[d x] + 3675 a^3 d x \operatorname{Cosh}[2 c+d x] + 2205 a^3 d x \operatorname{Cosh}[2 c+3 d x] + 2205 a^3 d x \operatorname{Cosh}[4 c+3 d x] + 735 a^3 d x \operatorname{Cosh}[4 c+5 d x] + \\
& 735 a^3 d x \operatorname{Cosh}[6 c+5 d x] + 105 a^3 d x \operatorname{Cosh}[6 c+7 d x] + 105 a^3 d x \operatorname{Cosh}[8 c+7 d x] - 4200 a^3 \operatorname{Sinh}[d x] + 3360 a^2 b \operatorname{Sinh}[d x] + \\
& 840 a b^2 \operatorname{Sinh}[d x] - 560 b^3 \operatorname{Sinh}[d x] + 3150 a^3 \operatorname{Sinh}[2 c+d x] - 3990 a^2 b \operatorname{Sinh}[2 c+d x] - 2100 a b^2 \operatorname{Sinh}[2 c+d x] - \\
& 1120 b^3 \operatorname{Sinh}[2 c+d x] - 3150 a^3 \operatorname{Sinh}[2 c+3 d x] + 1890 a^2 b \operatorname{Sinh}[2 c+3 d x] + 504 a b^2 \operatorname{Sinh}[2 c+3 d x] + \\
& 336 b^3 \operatorname{Sinh}[2 c+3 d x] + 1260 a^3 \operatorname{Sinh}[4 c+3 d x] - 2520 a^2 b \operatorname{Sinh}[4 c+3 d x] - 1260 a b^2 \operatorname{Sinh}[4 c+3 d x] - \\
& 1260 a^3 \operatorname{Sinh}[4 c+5 d x] + 840 a^2 b \operatorname{Sinh}[4 c+5 d x] + 588 a b^2 \operatorname{Sinh}[4 c+5 d x] + 112 b^3 \operatorname{Sinh}[4 c+5 d x] + 210 a^3 \operatorname{Sinh}[6 c+5 d x] - \\
& 630 a^2 b \operatorname{Sinh}[6 c+5 d x] - 210 a^3 \operatorname{Sinh}[6 c+7 d x] + 210 a^2 b \operatorname{Sinh}[6 c+7 d x] + 84 a b^2 \operatorname{Sinh}[6 c+7 d x] + 16 b^3 \operatorname{Sinh}[6 c+7 d x])
\end{aligned}$$

Problem 128: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$a^3 x + \frac{b (3 a^2 + 3 a b + b^2) \operatorname{Tanh}[c + d x]}{d} - \frac{b^2 (3 a + 2 b) \operatorname{Tanh}[c + d x]^3}{3 d} + \frac{b^3 \operatorname{Tanh}[c + d x]^5}{5 d}$$

Result (type 3, 268 leaves):

$$\frac{1}{480 d} \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^5$$

$$\left(150 a^3 d x \operatorname{Cosh}[d x] + 150 a^3 d x \operatorname{Cosh}[2 c + d x] + 75 a^3 d x \operatorname{Cosh}[2 c + 3 d x] + 75 a^3 d x \operatorname{Cosh}[4 c + 3 d x] + 15 a^3 d x \operatorname{Cosh}[4 c + 5 d x] + \right.$$

$$15 a^3 d x \operatorname{Cosh}[6 c + 5 d x] + 540 a^2 b \operatorname{Sinh}[d x] + 420 a b^2 \operatorname{Sinh}[d x] + 160 b^3 \operatorname{Sinh}[d x] - 360 a^2 b \operatorname{Sinh}[2 c + d x] - 180 a b^2 \operatorname{Sinh}[2 c + d x] +$$

$$360 a^2 b \operatorname{Sinh}[2 c + 3 d x] + 300 a b^2 \operatorname{Sinh}[2 c + 3 d x] + 80 b^3 \operatorname{Sinh}[2 c + 3 d x] - 90 a^2 b \operatorname{Sinh}[4 c + 3 d x] +$$

$$\left. 90 a^2 b \operatorname{Sinh}[4 c + 5 d x] + 60 a b^2 \operatorname{Sinh}[4 c + 5 d x] + 16 b^3 \operatorname{Sinh}[4 c + 5 d x] \right)$$

Problem 130: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^2 (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 61 leaves, 4 steps):

$$a^3 x - \frac{(a + b)^3 \operatorname{Coth}[c + d x]}{d} - \frac{b^2 (3 a + 2 b) \operatorname{Tanh}[c + d x]}{d} + \frac{b^3 \operatorname{Tanh}[c + d x]^3}{3 d}$$

Result (type 3, 126 leaves):

$$\left(8 (a \operatorname{Cosh}[c + d x] + b \operatorname{Sech}[c + d x])^3 \right.$$

$$\left. \left(3 a^3 d x \operatorname{Cosh}[c + d x]^3 - b^3 \operatorname{Sech}[c] \operatorname{Sinh}[d x] + \operatorname{Cosh}[c + d x]^2 \left(3 (a + b)^3 \operatorname{Coth}[c + d x] \operatorname{Csch}[c] - b^2 (9 a + 5 b) \operatorname{Sech}[c] \right) \operatorname{Sinh}[d x] - \right. \right.$$

$$\left. \left. b^3 \operatorname{Cosh}[c + d x] \operatorname{Tanh}[c] \right) \right) / \left(3 d (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3 \right)$$

Problem 131: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^3 (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$- \frac{(a + b)^3 \operatorname{Csch}[c + d x]^2}{2 d} + \frac{b^2 (3 a + 2 b) \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{d} + \frac{(a - 2 b) (a + b)^2 \operatorname{Log}[\operatorname{Sinh}[c + d x]]}{d} - \frac{b^3 \operatorname{Sech}[c + d x]^2}{2 d}$$

Result (type 3, 174 leaves):

$$-\frac{1}{2d} \operatorname{Csch}[2(c+dx)]^2 \left(2a^3 + 6a^2b + 6ab^2 + 2(a^3 + 3a^2b + 3ab^2 + 2b^3) \operatorname{Cosh}[2(c+dx)] + 3ab^2 \operatorname{Log}[\operatorname{Cosh}[c+dx]] + 2b^3 \operatorname{Log}[\operatorname{Cosh}[c+dx]] + a^3 \operatorname{Log}[\operatorname{Sinh}[c+dx]] - 3ab^2 \operatorname{Log}[\operatorname{Sinh}[c+dx]] - 2b^3 \operatorname{Log}[\operatorname{Sinh}[c+dx]] - \operatorname{Cosh}[4(c+dx)] \left(b^2(3a+2b) \operatorname{Log}[\operatorname{Cosh}[c+dx]] + (a-2b)(a+b)^2 \operatorname{Log}[\operatorname{Sinh}[c+dx]] \right) \right)$$

Problem 132: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c+dx]^4 (a+b \operatorname{Sech}[c+dx]^2)^3 dx$$

Optimal (type 3, 60 leaves, 4 steps):

$$a^3 x - \frac{(a-2b)(a+b)^2 \operatorname{Coth}[c+dx]}{d} - \frac{(a+b)^3 \operatorname{Coth}[c+dx]^3}{3d} + \frac{b^3 \operatorname{Tanh}[c+dx]}{d}$$

Result (type 3, 343 leaves):

$$\frac{1}{96d} \operatorname{Csch}[c] \operatorname{Csch}[c+dx]^3 \operatorname{Sech}[c] \operatorname{Sech}[c+dx] \left(6a^3 dx \operatorname{Cosh}[2dx] - 3a^3 dx \operatorname{Cosh}[2(c+2dx)] - 6a^3 dx \operatorname{Cosh}[4c+2dx] + 3a^3 dx \operatorname{Cosh}[6c+4dx] - 18a^2 b \operatorname{Sinh}[2c] - 36a^2 b \operatorname{Sinh}[2c] - 4a^3 \operatorname{Sinh}[2dx] + 6a^2 b \operatorname{Sinh}[2dx] + 24a^2 b \operatorname{Sinh}[2dx] + 32b^3 \operatorname{Sinh}[2dx] - 16a^3 \operatorname{Sinh}[2(c+dx)] - 12a^2 b \operatorname{Sinh}[2(c+dx)] + 24a^2 b \operatorname{Sinh}[2(c+dx)] + 8b^3 \operatorname{Sinh}[2(c+dx)] + 8a^3 \operatorname{Sinh}[4(c+dx)] + 6a^2 b \operatorname{Sinh}[4(c+dx)] - 12a^2 b \operatorname{Sinh}[4(c+dx)] - 4b^3 \operatorname{Sinh}[4(c+dx)] + 8a^3 \operatorname{Sinh}[2(c+2dx)] + 6a^2 b \operatorname{Sinh}[2(c+2dx)] - 12a^2 b \operatorname{Sinh}[2(c+2dx)] - 16b^3 \operatorname{Sinh}[2(c+2dx)] - 12a^3 \operatorname{Sinh}[4c+2dx] - 18a^2 b \operatorname{Sinh}[4c+2dx] \right)$$

Problem 134: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c+dx]^6 (a+b \operatorname{Sech}[c+dx]^2)^3 dx$$

Optimal (type 3, 69 leaves, 4 steps):

$$a^3 x - \frac{(a^3 + b^3) \operatorname{Coth}[c+dx]}{d} - \frac{(a-2b)(a+b)^2 \operatorname{Coth}[c+dx]^3}{3d} - \frac{(a+b)^3 \operatorname{Coth}[c+dx]^5}{5d}$$

Result (type 3, 303 leaves):

$$\frac{1}{480 d} \operatorname{Csch}[c] \operatorname{Csch}[c + d x]^5$$

$$\begin{aligned} & (-150 a^3 d x \operatorname{Cosh}[d x] + 150 a^3 d x \operatorname{Cosh}[2 c + d x] + 75 a^3 d x \operatorname{Cosh}[2 c + 3 d x] - 75 a^3 d x \operatorname{Cosh}[4 c + 3 d x] - 15 a^3 d x \operatorname{Cosh}[4 c + 5 d x] + \\ & 15 a^3 d x \operatorname{Cosh}[6 c + 5 d x] + 280 a^3 \operatorname{Sinh}[d x] + 180 a^2 b \operatorname{Sinh}[d x] + 60 a b^2 \operatorname{Sinh}[d x] + 160 b^3 \operatorname{Sinh}[d x] + 180 a^3 \operatorname{Sinh}[2 c + d x] - \\ & 180 a b^2 \operatorname{Sinh}[2 c + d x] - 140 a^3 \operatorname{Sinh}[2 c + 3 d x] + 60 a b^2 \operatorname{Sinh}[2 c + 3 d x] - 80 b^3 \operatorname{Sinh}[2 c + 3 d x] - 90 a^3 \operatorname{Sinh}[4 c + 3 d x] - \\ & 90 a^2 b \operatorname{Sinh}[4 c + 3 d x] + 46 a^3 \operatorname{Sinh}[4 c + 5 d x] + 18 a^2 b \operatorname{Sinh}[4 c + 5 d x] - 12 a b^2 \operatorname{Sinh}[4 c + 5 d x] + 16 b^3 \operatorname{Sinh}[4 c + 5 d x]) \end{aligned}$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + d x]^2)^4 dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$a^4 x + \frac{b(2a+b)(2a^2+2ab+b^2)\operatorname{Tanh}[c+dx]}{d} - \frac{b^2(6a^2+8ab+3b^2)\operatorname{Tanh}[c+dx]^3}{3d} + \frac{b^3(4a+3b)\operatorname{Tanh}[c+dx]^5}{5d} - \frac{b^4\operatorname{Tanh}[c+dx]^7}{7d}$$

Result (type 3, 455 leaves):

$$\frac{1}{13440 d} \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^7$$

$$\begin{aligned} & (3675 a^4 d x \operatorname{Cosh}[d x] + 3675 a^4 d x \operatorname{Cosh}[2 c + d x] + 2205 a^4 d x \operatorname{Cosh}[2 c + 3 d x] + 2205 a^4 d x \operatorname{Cosh}[4 c + 3 d x] + 735 a^4 d x \operatorname{Cosh}[4 c + 5 d x] + \\ & 735 a^4 d x \operatorname{Cosh}[6 c + 5 d x] + 105 a^4 d x \operatorname{Cosh}[6 c + 7 d x] + 105 a^4 d x \operatorname{Cosh}[8 c + 7 d x] + 16800 a^3 b \operatorname{Sinh}[d x] + 18480 a^2 b^2 \operatorname{Sinh}[d x] + \\ & 11200 a b^3 \operatorname{Sinh}[d x] + 3360 b^4 \operatorname{Sinh}[d x] - 12600 a^3 b \operatorname{Sinh}[2 c + d x] - 10920 a^2 b^2 \operatorname{Sinh}[2 c + d x] - 4480 a b^3 \operatorname{Sinh}[2 c + d x] + \\ & 12600 a^3 b \operatorname{Sinh}[2 c + 3 d x] + 15120 a^2 b^2 \operatorname{Sinh}[2 c + 3 d x] + 9408 a b^3 \operatorname{Sinh}[2 c + 3 d x] + 2016 b^4 \operatorname{Sinh}[2 c + 3 d x] - 5040 a^3 b \operatorname{Sinh}[4 c + 3 d x] - \\ & 2520 a^2 b^2 \operatorname{Sinh}[4 c + 3 d x] + 5040 a^3 b \operatorname{Sinh}[4 c + 5 d x] + 5880 a^2 b^2 \operatorname{Sinh}[4 c + 5 d x] + 3136 a b^3 \operatorname{Sinh}[4 c + 5 d x] + 672 b^4 \operatorname{Sinh}[4 c + 5 d x] - \\ & 840 a^3 b \operatorname{Sinh}[6 c + 5 d x] + 840 a^3 b \operatorname{Sinh}[6 c + 7 d x] + 840 a^2 b^2 \operatorname{Sinh}[6 c + 7 d x] + 448 a b^3 \operatorname{Sinh}[6 c + 7 d x] + 96 b^4 \operatorname{Sinh}[6 c + 7 d x]) \end{aligned}$$

Problem 137: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + d x]^2)^5 dx$$

Optimal (type 3, 163 leaves, 4 steps):

$$a^5 x + \frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\operatorname{Tanh}[c+dx]}{d} - \frac{b^2(10a^3+20a^2b+15ab^2+4b^3)\operatorname{Tanh}[c+dx]^3}{3d} +$$

$$\frac{b^3(10a^2+15ab+6b^2)\operatorname{Tanh}[c+dx]^5}{5d} - \frac{b^4(5a+4b)\operatorname{Tanh}[c+dx]^7}{7d} + \frac{b^5\operatorname{Tanh}[c+dx]^9}{9d}$$

Result (type 3, 724 leaves):

$$\begin{aligned}
& \frac{32 a^5 x \operatorname{Cosh}[c+d x]^{10} (a+b \operatorname{Sech}[c+d x]^2)^5}{(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5} + \frac{32 \operatorname{Cosh}[c+d x]^4 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 (45 a b^4 \operatorname{Sinh}[c] + 8 b^5 \operatorname{Sinh}[c])}{63 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5} + \\
& \frac{64 \operatorname{Cosh}[c+d x]^6 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 (105 a^2 b^3 \operatorname{Sinh}[c] + 45 a b^4 \operatorname{Sinh}[c] + 8 b^5 \operatorname{Sinh}[c])}{105 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5} + \\
& \left(64 \operatorname{Cosh}[c+d x]^8 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 (525 a^3 b^2 \operatorname{Sinh}[c] + 420 a^2 b^3 \operatorname{Sinh}[c] + 180 a b^4 \operatorname{Sinh}[c] + 32 b^5 \operatorname{Sinh}[c]) \right) / \\
& \left(315 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5 \right) + \frac{32 b^5 \operatorname{Cosh}[c+d x] \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 \operatorname{Sinh}[d x]}{9 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5} + \\
& \frac{32 \operatorname{Cosh}[c+d x]^3 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 (45 a b^4 \operatorname{Sinh}[d x] + 8 b^5 \operatorname{Sinh}[d x])}{63 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5} + \\
& \left(64 \operatorname{Cosh}[c+d x]^5 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 (105 a^2 b^3 \operatorname{Sinh}[d x] + 45 a b^4 \operatorname{Sinh}[d x] + 8 b^5 \operatorname{Sinh}[d x]) \right) / \left(105 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5 \right) + \\
& \left(64 \operatorname{Cosh}[c+d x]^7 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 (525 a^3 b^2 \operatorname{Sinh}[d x] + 420 a^2 b^3 \operatorname{Sinh}[d x] + 180 a b^4 \operatorname{Sinh}[d x] + 32 b^5 \operatorname{Sinh}[d x]) \right) / \\
& \left(315 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5 \right) + \left(32 \operatorname{Cosh}[c+d x]^9 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^5 \right. \\
& \left. (1575 a^4 b \operatorname{Sinh}[d x] + 2100 a^3 b^2 \operatorname{Sinh}[d x] + 1680 a^2 b^3 \operatorname{Sinh}[d x] + 720 a b^4 \operatorname{Sinh}[d x] + 128 b^5 \operatorname{Sinh}[d x]) \right) / \\
& \left(315 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5 \right) + \frac{32 b^5 \operatorname{Cosh}[c+d x]^2 (a+b \operatorname{Sech}[c+d x]^2)^5 \operatorname{Tanh}[c]}{9 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^5}
\end{aligned}$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+d x]^5}{a+b \operatorname{Sech}[c+d x]^2} dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$-\frac{(a+2 b) \operatorname{Log}[\operatorname{Cosh}[c+d x]]}{b^2 d} + \frac{(a+b)^2 \operatorname{Log}[b+a \operatorname{Cosh}[c+d x]^2]}{2 a b^2 d} - \frac{\operatorname{Sech}[c+d x]^2}{2 b d}$$

Result (type 3, 180 leaves):

$$\begin{aligned}
& -\frac{1}{8 a b^2 d (a+b \operatorname{Sech}[c+d x]^2)} (a+2 b+a \operatorname{Cosh}[2(c+d x)]) \left(2 a b+2 a(a+2 b) \operatorname{Log}[\operatorname{Cosh}[c+d x]] - \right. \\
& \left. a^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]] - 2 a b \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]] - b^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]] \right) + \\
& \left. \operatorname{Cosh}[2(c+d x)] \left(2 a(a+2 b) \operatorname{Log}[\operatorname{Cosh}[c+d x]] - (a+b)^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]] \right) \right) \operatorname{Sech}[c+d x]^4
\end{aligned}$$

Problem 139: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[c + d x]^4}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 59 leaves, 6 steps):

$$\frac{x}{a} - \frac{(a + b)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{a b^{3/2} d} + \frac{\text{Tanh}[c + d x]}{b d}$$

Result (type 3, 196 leaves):

$$\left((a + 2 b + a \text{Cosh}[2 (c + d x)]) \text{Sech}[c + d x]^2 \right. \\ \left. \left((a + b)^2 \text{ArcTanh}\left[\frac{\text{Sech}[d x] (\text{Cosh}[2 c] - \text{Sinh}[2 c]) ((a + 2 b) \text{Sinh}[d x] - a \text{Sinh}[2 c + d x])}{2 \sqrt{a + b} \sqrt{b (\text{Cosh}[c] - \text{Sinh}[c])^4}} \right] (-\text{Cosh}[2 c] + \text{Sinh}[2 c]) + \right. \right. \\ \left. \left. \sqrt{a + b} \sqrt{b (\text{Cosh}[c] - \text{Sinh}[c])^4} (b d x + a \text{Sech}[c] \text{Sech}[c + d x] \text{Sinh}[d x]) \right) \right) / \\ \left(2 a b \sqrt{a + b} d (a + b \text{Sech}[c + d x]^2) \sqrt{b (\text{Cosh}[c] - \text{Sinh}[c])^4} \right)$$

Problem 141: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[c + d x]^2}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$\frac{x}{a} - \frac{\sqrt{a + b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{a \sqrt{b} d}$$

Result (type 3, 174 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^2 \left(\sqrt{a+b} dx \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} + \right. \right. \\ \left. \left. (a+b) \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a+2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} \right] (-\operatorname{Cosh}[2c] + \operatorname{Sinh}[2c]) \right) \right) \Bigg) / \\ \left(2a\sqrt{a+b} d (a+b \operatorname{Sech}[c + dx]^2) \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right)$$

Problem 143: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a + b \operatorname{Sech}[c + dx]^2} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{x}{a} - \frac{\sqrt{b} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}} \right]}{a\sqrt{a+b}d}$$

Result (type 3, 172 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^2 \left(\sqrt{a+b} dx \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} + \right. \right. \\ \left. \left. b \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a+2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} \right] (-\operatorname{Cosh}[2c] + \operatorname{Sinh}[2c]) \right) \right) \Bigg) / \\ \left(2a\sqrt{a+b} d (a+b \operatorname{Sech}[c + dx]^2) \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right)$$

Problem 145: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + dx]^2}{a + b \operatorname{Sech}[c + dx]^2} dx$$

Optimal (type 3, 62 leaves, 6 steps):

$$\frac{x}{a} - \frac{b^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}} \right]}{a(a+b)^{3/2}d} - \frac{\operatorname{Coth}[c + dx]}{(a+b)d}$$

Result (type 3, 193 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^2 \right. \\ \left. \left(b^2 \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} \right] (-\operatorname{Cosh}[2c] + \operatorname{Sinh}[2c]) + \right. \right. \\ \left. \left. \sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} ((a+b) dx + a \operatorname{CsCh}[c] \operatorname{CsCh}[c + dx] \operatorname{Sinh}[dx]) \right) \right) \Bigg) / \\ \left(2a(a+b)^{3/2} d (a+b \operatorname{Sech}[c + dx]^2) \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right)$$

Problem 147: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + dx]^4}{a + b \operatorname{Sech}[c + dx]^2} dx$$

Optimal (type 3, 87 leaves, 7 steps):

$$\frac{x}{a} - \frac{b^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{a(a+b)^{5/2} d} - \frac{(a+2b) \operatorname{Coth}[c + dx]}{(a+b)^2 d} - \frac{\operatorname{Coth}[c + dx]^3}{3(a+b)d}$$

Result (type 3, 581 leaves):

$$\begin{aligned}
& \frac{x (a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Sech}[c + dx]^2}{2a (a + b \operatorname{Sech}[c + dx]^2)} - \\
& \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Coth}[c] \operatorname{Csch}[c + dx]^2 \operatorname{Sech}[c + dx]^2}{6(a + b)d (a + b \operatorname{Sech}[c + dx]^2)} + \left((a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Sech}[c + dx]^2 \right. \\
& \left. \left(\left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \\
& \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) / \left(2a\sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \\
& \left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
& \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \left. \right) / \left(2a\sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \left. \right) / \\
& \left((a + b)^2 (a + b \operatorname{Sech}[c + dx]^2) \right) + \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Csch}[c] \operatorname{Csch}[c + dx]^3 \operatorname{Sech}[c + dx]^2 \operatorname{Sinh}[dx]}{6(a + b)d (a + b \operatorname{Sech}[c + dx]^2)} + \\
& \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Csch}[c] \operatorname{Csch}[c + dx] \operatorname{Sech}[c + dx]^2 (4a \operatorname{Sinh}[dx] + 7b \operatorname{Sinh}[dx])}{6(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)}
\end{aligned}$$

Problem 149: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c + dx]^4}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$\frac{x}{a^2} + \frac{(a - 2b) \sqrt{a + b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a + b}}\right]}{2a^2 b^{3/2} d} - \frac{(a + b) \operatorname{Tanh}[c + dx]}{2abd (a + b - b \operatorname{Tanh}[c + dx]^2)}$$

Result (type 3, 228 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^4 \right. \\ \left. \left(2x (a + 2b + a \operatorname{Cosh}[2(c + dx)]) + \left((a^2 - ab - 2b^2) \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] \right) \right. \right. \\ \left. \left. (a + 2b + a \operatorname{Cosh}[2(c + dx)]) (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \right) / \left(b\sqrt{a+b} d \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right) + \\ \left. \left. \frac{(a+b) \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{bd} \right) \right) / \left(8a^2 (a + b \operatorname{Sech}[c + dx]^2)^2 \right)$$

Problem 151: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c + dx]^2}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{x}{a^2} - \frac{(a + 2b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{2a^2 \sqrt{b} \sqrt{a+b} d} - \frac{\operatorname{Tanh}[c + dx]}{2ad (a + b - b \operatorname{Tanh}[c + dx]^2)}$$

Result (type 3, 372 leaves):

$$\frac{\left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \right.}{\left(16x + \left((a^3 - 6a^2b - 24ab^2 - 16b^3) \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right] \right. \right.} \\
\left. \left. (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \right) / \left(b (a+b)^{3/2} d \sqrt{b} (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4 \right) + \left. \frac{(a^2 + 8ab + 8b^2) \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{b (a+b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) / \left(64a^2 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \left. \left(a + 2b + a \operatorname{Cosh}[2c + 2dx] \right)^2 \operatorname{Sech}[c + dx]^4 \left(- \frac{(a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{8b^{3/2} (a+b)^{3/2} d} + \frac{a \operatorname{Sinh}[2(c + dx)]}{8b (a+b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) \right)}{8 (a + b \operatorname{Sech}[c + dx]^2)^2}$$

Problem 153: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{2a^2 (a+b)^{3/2} d} - \frac{b \operatorname{Tanh}[c + dx]}{2a (a+b) d (a + b - b \operatorname{Tanh}[c + dx]^2)}$$

Result (type 3, 221 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^4 \right. \\ \left. \left(2x(a + 2b + a \operatorname{Cosh}[2(c + dx)]) - \left(b(3a + 2b) \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} \right] \right) \right. \right. \\ \left. \left. (a + 2b + a \operatorname{Cosh}[2(c + dx)]) (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \right) / \left((a + b)^{3/2} d \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + \\ \left. \frac{b \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{(a + b)d} \right) / \left(8a^2 (a + b \operatorname{Sech}[c + dx]^2)^2 \right)$$

Problem 155: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + dx]^2}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 121 leaves, 7 steps):

$$\frac{x}{a^2} - \frac{b^{3/2} (5a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}}\right]}{2a^2 (a + b)^{5/2} d} - \frac{(2a - b) \operatorname{Coth}[c + dx]}{2a (a + b)^2 d} - \frac{b \operatorname{Coth}[c + dx]}{2a (a + b) d (a + b - b \operatorname{Tanh}[c + dx]^2)}$$

Result (type 3, 268 leaves):

$$\frac{1}{8 (a + b \operatorname{Sech}[c + dx]^2)^2} (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^4 \\ \left(\frac{2x(a + 2b + a \operatorname{Cosh}[2(c + dx)])}{a^2} - \left(b^2 (5a + 2b) \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} \right] \right) \right. \\ \left. (a + 2b + a \operatorname{Cosh}[2(c + dx)]) (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) / \left(a^2 (a + b)^{5/2} d \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + \\ \left. \frac{2(a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Csch}[c] \operatorname{Csch}[c + dx] \operatorname{Sinh}[dx]}{(a + b)^2 d} + \frac{b^2 \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{a^2 (a + b)^2 d} \right)$$

Problem 157: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + d x]^4}{(a + b \operatorname{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$\frac{x}{a^2} - \frac{b^{5/2} (7a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{2 a^2 (a + b)^{7/2} d} - \frac{(2 a^2 + 6 a b - b^2) \operatorname{Coth}[c + d x]}{2 a (a + b)^3 d} - \frac{(2 a - 3 b) \operatorname{Coth}[c + d x]^3}{6 a (a + b)^2 d} - \frac{b \operatorname{Coth}[c + d x]^3}{2 a (a + b) d (a + b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 685 leaves):

$$\begin{aligned} & \frac{x (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Sech}[c + d x]^4}{4 a^2 (a + b \operatorname{Sech}[c + d x]^2)^2} - \\ & \frac{(a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Coth}[c] \operatorname{Csch}[c + d x]^2 \operatorname{Sech}[c + d x]^4}{12 (a + b)^2 d (a + b \operatorname{Sech}[c + d x]^2)^2} + \left((7 a + 2 b) (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \right. \\ & \left. \operatorname{Sech}[c + d x]^4 \left(\left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \right. \right. \\ & \left. \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \right) \operatorname{Cosh}[2 c] \right) / \left(8 a^2 \sqrt{a + b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) - \right. \\ & \left. \left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \right. \\ & \left. \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \right) \operatorname{Sinh}[2 c] \right) / \left(8 a^2 \sqrt{a + b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) \right) / \\ & \left((a + b)^3 (a + b \operatorname{Sech}[c + d x]^2)^2 \right) + \frac{(a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Csch}[c] \operatorname{Csch}[c + d x]^3 \operatorname{Sech}[c + d x]^4 \operatorname{Sinh}[d x]}{12 (a + b)^2 d (a + b \operatorname{Sech}[c + d x]^2)^2} + \\ & \frac{(a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Csch}[c] \operatorname{Csch}[c + d x] \operatorname{Sech}[c + d x]^4 (2 a \operatorname{Sinh}[d x] + 5 b \operatorname{Sinh}[d x])}{6 (a + b)^3 d (a + b \operatorname{Sech}[c + d x]^2)^2} + \\ & \frac{(a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]) \operatorname{Sech}[2 c] \operatorname{Sech}[c + d x]^4 (a b^3 \operatorname{Sinh}[2 c] + 2 b^4 \operatorname{Sinh}[2 c] - a b^3 \operatorname{Sinh}[2 d x])}{8 a^2 (a + b)^3 d (a + b \operatorname{Sech}[c + d x]^2)^2} \end{aligned}$$

Problem 158: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c + d x]^6}{(a + b \operatorname{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 148 leaves, 7 steps):

$$\frac{x}{a^3} - \frac{\sqrt{a+b} (3a^2 - 4ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8a^3 b^{5/2} d} - \frac{(a+b) \operatorname{Tanh}[c+dx]^3}{4abd(a+b-b \operatorname{Tanh}[c+dx]^2)^2} + \frac{(3a-4b)(a+b) \operatorname{Tanh}[c+dx]}{8a^2 b^2 d(a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 754 leaves):

$$\frac{1}{(a+b \operatorname{Sech}[c+dx]^2)^3} (3a^3 - a^2 b + 4ab^2 + 8b^3) (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6$$

$$\left(\left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \left(64a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) -$$

$$\left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / \left(64a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \Bigg) +$$

$$\frac{1}{128a^3 b^2 d (a+b \operatorname{Sech}[c+dx]^2)^3} (a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^6$$

$$(24a^2 b^2 dx \operatorname{Cosh}[2c] + 64a^3 b^3 dx \operatorname{Cosh}[2c] + 64b^4 dx \operatorname{Cosh}[2c] + 16a^2 b^2 dx \operatorname{Cosh}[2dx] + 32ab^3 dx \operatorname{Cosh}[2dx] +$$

$$16a^2 b^2 dx \operatorname{Cosh}[4c+2dx] + 32ab^3 dx \operatorname{Cosh}[4c+2dx] + 4a^2 b^2 dx \operatorname{Cosh}[2c+4dx] + 4a^2 b^2 dx \operatorname{Cosh}[6c+4dx] -$$

$$9a^4 \operatorname{Sinh}[2c] - 15a^3 b \operatorname{Sinh}[2c] + 18a^2 b^2 \operatorname{Sinh}[2c] + 72ab^3 \operatorname{Sinh}[2c] + 48b^4 \operatorname{Sinh}[2c] + 9a^4 \operatorname{Sinh}[2dx] +$$

$$13a^3 b \operatorname{Sinh}[2dx] - 28a^2 b^2 \operatorname{Sinh}[2dx] - 32ab^3 \operatorname{Sinh}[2dx] - 3a^4 \operatorname{Sinh}[4c+2dx] + a^3 b \operatorname{Sinh}[4c+2dx] +$$

$$20a^2 b^2 \operatorname{Sinh}[4c+2dx] + 16ab^3 \operatorname{Sinh}[4c+2dx] + 3a^4 \operatorname{Sinh}[2c+4dx] - 3a^3 b \operatorname{Sinh}[2c+4dx] - 6a^2 b^2 \operatorname{Sinh}[2c+4dx])$$

Problem 160: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+dx]^4}{(a+b \operatorname{Sech}[c+dx]^2)^3} dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$\frac{x}{a^3} + \frac{(a^2 - 4ab - 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8a^3 b^{3/2} \sqrt{a+b} d} - \frac{(a+b) \operatorname{Tanh}[c+dx]}{4abd(a+b-b \operatorname{Tanh}[c+dx]^2)^2} + \frac{(a-4b) \operatorname{Tanh}[c+dx]}{8a^2 b d(a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 1730 leaves):

$$\begin{aligned}
& \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \right. \\
& \left. \left(\frac{(3a^2 + 8ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{a\sqrt{b} (3a^2 + 16ab + 16b^2 + 3a(a+2b) \operatorname{Cosh}[2(c+dx)]) \operatorname{Sinh}[2(c+dx)]}{(a+b)^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])^2} \right) \right) / \\
& (1024 b^{5/2} d (a + b \operatorname{Sech}[c + dx]^2)^3) - \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \right. \\
& \left. \left(-\frac{3a(a+2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{\sqrt{b} (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \operatorname{Cosh}[2(c+dx)]) \operatorname{Sinh}[2(c+dx)]}{(a+b)^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])^2} \right) \right) / \\
& (2048 b^{5/2} d (a + b \operatorname{Sech}[c + dx]^2)^3) + \frac{1}{32 (a + b \operatorname{Sech}[c + dx]^2)^3} \\
& (a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \left(\frac{1}{(a+b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \right. \\
& \left. \left(\left(\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \\
& \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) / (64a^3b^2\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}) - \\
& \left(\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
& \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \right) / (64a^3b^2\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}) \Big) + \\
& \frac{1}{128a^3b^2(a+b)^2d(a+2b+a \operatorname{Cosh}[2c+2dx])^2} \operatorname{Sech}[2c] (768a^4b^2dx \operatorname{Cosh}[2c] + 3584a^3b^3dx \operatorname{Cosh}[2c] + 6912a^2b^4dx \operatorname{Cosh}[2c] + \\
& 6144a^5b^5dx \operatorname{Cosh}[2c] + 2048b^6dx \operatorname{Cosh}[2c] + 512a^4b^2dx \operatorname{Cosh}[2dx] + 2048a^3b^3dx \operatorname{Cosh}[2dx] + 2560a^2b^4dx \operatorname{Cosh}[2dx] + \\
& 1024a^5b^5dx \operatorname{Cosh}[2dx] + 512a^4b^2dx \operatorname{Cosh}[4c+2dx] + 2048a^3b^3dx \operatorname{Cosh}[4c+2dx] + 2560a^2b^4dx \operatorname{Cosh}[4c+2dx] + \\
& 1024a^5b^5dx \operatorname{Cosh}[4c+2dx] + 128a^4b^2dx \operatorname{Cosh}[2c+4dx] + 256a^3b^3dx \operatorname{Cosh}[2c+4dx] + 128a^2b^4dx \operatorname{Cosh}[2c+4dx] + \\
& 128a^4b^2dx \operatorname{Cosh}[6c+4dx] + 256a^3b^3dx \operatorname{Cosh}[6c+4dx] + 128a^2b^4dx \operatorname{Cosh}[6c+4dx] - 9a^6 \operatorname{Sinh}[2c] + 12a^5b \operatorname{Sinh}[2c] + \\
& 684a^4b^2 \operatorname{Sinh}[2c] + 2880a^3b^3 \operatorname{Sinh}[2c] + 5280a^2b^4 \operatorname{Sinh}[2c] + 4608ab^5 \operatorname{Sinh}[2c] + 1536b^6 \operatorname{Sinh}[2c] + 9a^6 \operatorname{Sinh}[2dx] - \\
& 14a^5b \operatorname{Sinh}[2dx] - 608a^4b^2 \operatorname{Sinh}[2dx] - 2112a^3b^3 \operatorname{Sinh}[2dx] - 2560a^2b^4 \operatorname{Sinh}[2dx] - 1024ab^5 \operatorname{Sinh}[2dx] - 3a^6 \operatorname{Sinh}[4c+2dx] + \\
& 10a^5b \operatorname{Sinh}[4c+2dx] + 304a^4b^2 \operatorname{Sinh}[4c+2dx] + 1056a^3b^3 \operatorname{Sinh}[4c+2dx] + 1280a^2b^4 \operatorname{Sinh}[4c+2dx] + 512ab^5 \operatorname{Sinh}[4c+2dx] +
\end{aligned}$$

$$\left. \begin{aligned} & 3 a^6 \operatorname{Sinh}[2 c+4 d x]-12 a^5 b \operatorname{Sinh}[2 c+4 d x]-204 a^4 b^2 \operatorname{Sinh}[2 c+4 d x]-384 a^3 b^3 \operatorname{Sinh}[2 c+4 d x]-192 a^2 b^4 \operatorname{Sinh}[2 c+4 d x] \end{aligned} \right) - \\ & \frac{1}{2048 b^2 (a+b)^2 d (a+b \operatorname{Sech}[c+d x]^2)^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6 \\ & \left(\frac{6 a^2 \operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[d x](\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c])((a+2 b) \operatorname{Sinh}[d x]-a \operatorname{Sinh}[2 c+d x])}{2 \sqrt{a+b} \sqrt{b}(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4}\right]}{\sqrt{a+b} \sqrt{b}(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4}\right) (\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c]) + \\ & (a \operatorname{Sech}[2 c]((-9 a^4-16 a^3 b+48 a^2 b^2+128 a b^3+64 b^4) \operatorname{Sinh}[2 d x]+ \\ & a(-3 a^3+2 a^2 b+24 a b^2+16 b^3) \operatorname{Sinh}[2(c+2 d x)]+(3 a^4-64 a^2 b^2-128 a b^3-64 b^4) \operatorname{Sinh}[4 c+2 d x]) + \\ & (9 a^5+18 a^4 b-64 a^3 b^2-256 a^2 b^3-320 a b^4-128 b^5) \operatorname{Tanh}[2 c]) / \left(a^2(a+2 b+a \operatorname{Cosh}[2(c+d x)])^2\right) \end{aligned} \right)$$

Problem 162: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+d x]^2}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$\frac{x}{a^3} - \frac{(3 a^2+12 a b+8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 a^3 \sqrt{b} (a+b)^{3/2} d} - \frac{\operatorname{Tanh}[c+d x]}{4 a d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} - \frac{(3 a+4 b) \operatorname{Tanh}[c+d x]}{8 a^2 (a+b) d (a+b-b \operatorname{Tanh}[c+d x]^2)}$$

Result (type 3, 1730 leaves):

$$- \left(\left((a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6 \right. \right. \\ \left. \left. \frac{\left((3 a^2+8 a b+8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right] - a \sqrt{b} (3 a^2+16 a b+16 b^2+3 a(a+2 b) \operatorname{Cosh}[2(c+d x)]) \operatorname{Sinh}[2(c+d x)] \right)}{(a+b)^{5/2} (a+b)^2 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2} \right) \right) / \\ \left. \left(1024 b^{5/2} d (a+b \operatorname{Sech}[c+d x]^2)^3 \right) - \left((a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6 \right) \right)$$

$$\left(-\frac{3a(a+2b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{\sqrt{b}(3a^3+14a^2b+24ab^2+16b^3+a(3a^2+4ab+4b^2)\operatorname{Cosh}[2(c+dx)])\operatorname{Sinh}[2(c+dx)]}{(a+b)^2(a+2b+a\operatorname{Cosh}[2(c+dx)])^2} \right) \Bigg/$$

$$(2048b^{5/2}d(a+b\operatorname{Sech}[c+dx]^2)^3) + \frac{1}{32(a+b\operatorname{Sech}[c+dx]^2)^3}$$

$$(a+2b+a\operatorname{Cosh}[2c+2dx])^3\operatorname{Sech}[c+dx]^6 \left(\frac{1}{(a+b)^2}(3a^5-10a^4b+80a^3b^2+480a^2b^3+640ab^4+256b^5) \right.$$

$$\left. \left(\left(i\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i\operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i\operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a\operatorname{Sinh}[dx]-2b\operatorname{Sinh}[dx]+a\operatorname{Sinh}[2c+dx])\operatorname{Cosh}[2c] \right) \Bigg/ (64a^3b^2\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}) - \right.$$

$$\left. \left(i\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i\operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i\operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a\operatorname{Sinh}[dx]-2b\operatorname{Sinh}[dx]+a\operatorname{Sinh}[2c+dx])\operatorname{Sinh}[2c] \right) \Bigg/ (64a^3b^2\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}) \right) +$$

$$\frac{1}{128a^3b^2(a+b)^2d(a+2b+a\operatorname{Cosh}[2c+2dx])^2} \operatorname{Sech}[2c] (768a^4b^2dx\operatorname{Cosh}[2c]+3584a^3b^3dx\operatorname{Cosh}[2c]+6912a^2b^4dx\operatorname{Cosh}[2c]+$$

$$6144a^5dx\operatorname{Cosh}[2c]+2048b^6dx\operatorname{Cosh}[2c]+512a^4b^2dx\operatorname{Cosh}[2dx]+2048a^3b^3dx\operatorname{Cosh}[2dx]+2560a^2b^4dx\operatorname{Cosh}[2dx]+$$

$$1024a^5dx\operatorname{Cosh}[2dx]+512a^4b^2dx\operatorname{Cosh}[4c+2dx]+2048a^3b^3dx\operatorname{Cosh}[4c+2dx]+2560a^2b^4dx\operatorname{Cosh}[4c+2dx]+$$

$$1024a^5dx\operatorname{Cosh}[4c+2dx]+128a^4b^2dx\operatorname{Cosh}[2c+4dx]+256a^3b^3dx\operatorname{Cosh}[2c+4dx]+128a^2b^4dx\operatorname{Cosh}[2c+4dx]+$$

$$128a^4b^2dx\operatorname{Cosh}[6c+4dx]+256a^3b^3dx\operatorname{Cosh}[6c+4dx]+128a^2b^4dx\operatorname{Cosh}[6c+4dx]-9a^6\operatorname{Sinh}[2c]+12a^5b\operatorname{Sinh}[2c]+$$

$$684a^4b^2\operatorname{Sinh}[2c]+2880a^3b^3\operatorname{Sinh}[2c]+5280a^2b^4\operatorname{Sinh}[2c]+4608a^5\operatorname{Sinh}[2c]+1536b^6\operatorname{Sinh}[2c]+9a^6\operatorname{Sinh}[2dx]-$$

$$14a^5b\operatorname{Sinh}[2dx]-608a^4b^2\operatorname{Sinh}[2dx]-2112a^3b^3\operatorname{Sinh}[2dx]-2560a^2b^4\operatorname{Sinh}[2dx]-1024a^5\operatorname{Sinh}[2dx]-3a^6\operatorname{Sinh}[4c+2dx]+$$

$$10a^5b\operatorname{Sinh}[4c+2dx]+304a^4b^2\operatorname{Sinh}[4c+2dx]+1056a^3b^3\operatorname{Sinh}[4c+2dx]+1280a^2b^4\operatorname{Sinh}[4c+2dx]+512a^5\operatorname{Sinh}[4c+2dx]+$$

$$3a^6\operatorname{Sinh}[2c+4dx]-12a^5b\operatorname{Sinh}[2c+4dx]-204a^4b^2\operatorname{Sinh}[2c+4dx]-384a^3b^3\operatorname{Sinh}[2c+4dx]-192a^2b^4\operatorname{Sinh}[2c+4dx]) \Bigg) +$$

$$\frac{1}{2048b^2(a+b)^2d(a+b\operatorname{Sech}[c+dx]^2)^3} (a+2b+a\operatorname{Cosh}[2c+2dx])^3$$

$$\operatorname{Sech}[c+dx]^6$$

$$\left(\frac{6a^2\operatorname{ArcTanh}\left[\frac{\operatorname{Sech}[dx](\operatorname{Cosh}[2c]-\operatorname{Sinh}[2c])(a+2b)\operatorname{Sinh}[dx]-a\operatorname{Sinh}[2c+dx]}{2\sqrt{a+b}\sqrt{b(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4}}\right]}{\sqrt{a+b}\sqrt{b(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4}} \right) (\operatorname{Cosh}[2c]-\operatorname{Sinh}[2c]) +$$

$$(a\operatorname{Sech}[2c]((-9a^4-16a^3b+48a^2b^2+128ab^3+64b^4)\operatorname{Sinh}[2dx]+$$

$$a(-3a^3+2a^2b+24ab^2+16b^3)\operatorname{Sinh}[2(c+2dx)]+(3a^4-64a^2b^2-128ab^3-64b^4)\operatorname{Sinh}[4c+2dx]) +$$

$$\left. (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \operatorname{Tanh}[2 c] \right/ \left(a^2 (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2 \right)$$

Problem 164: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Sech}[c + d x])^3} dx$$

Optimal (type 3, 146 leaves, 6 steps):

$$\frac{x}{a^3} - \frac{\sqrt{b} (15 a^2 + 20 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{8 a^3 (a + b)^{5/2} d} - \frac{b \operatorname{Tanh}[c + d x]}{4 a (a + b) d (a + b - b \operatorname{Tanh}[c + d x])^2} - \frac{b (7 a + 4 b) \operatorname{Tanh}[c + d x]}{8 a^2 (a + b)^2 d (a + b - b \operatorname{Tanh}[c + d x])^2}$$

Result (type 3, 597 leaves):

$$\begin{aligned} & \frac{x (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \operatorname{Sech}[c + d x]^6}{8 a^3 (a + b \operatorname{Sech}[c + d x])^3} + \frac{1}{(a + b)^2 (a + b \operatorname{Sech}[c + d x])^3} (15 a^2 + 20 a b + 8 b^2) (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \\ & \operatorname{Sech}[c + d x]^6 \left(\left(i b \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \right. \\ & \quad \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \right) \operatorname{Cosh}[2 c] \right) / \left(64 a^3 \sqrt{a + b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) - \\ & \left(i b \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \\ & \quad \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \right) \operatorname{Sinh}[2 c] \Big/ \left(64 a^3 \sqrt{a + b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) \Big) + \\ & \left((a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Sech}[2 c] \operatorname{Sech}[c + d x]^6 (9 a^2 b \operatorname{Sinh}[2 c] + 28 a b^2 \operatorname{Sinh}[2 c] + 16 b^3 \operatorname{Sinh}[2 c] - \right. \\ & \quad \left. 9 a^2 b \operatorname{Sinh}[2 d x] - 6 a b^2 \operatorname{Sinh}[2 d x]) \right) / \left(64 a^3 (a + b)^2 d (a + b \operatorname{Sech}[c + d x])^3 \right) + \\ & \left((a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]) \operatorname{Sech}[2 c] \operatorname{Sech}[c + d x]^6 (-a b^2 \operatorname{Sinh}[2 c] - 2 b^3 \operatorname{Sinh}[2 c] + a b^2 \operatorname{Sinh}[2 d x]) \right) / \\ & \left(16 a^3 (a + b) d (a + b \operatorname{Sech}[c + d x])^3 \right) \end{aligned}$$

Problem 165: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + d x]}{(a + b \operatorname{Sech}[c + d x])^3} dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$-\frac{b^3}{4 a^3 (a+b) d (b+a \operatorname{Cosh}[c+d x]^2)^2} + \frac{b^2 (3 a+2 b)}{2 a^3 (a+b)^2 d (b+a \operatorname{Cosh}[c+d x]^2)} + \frac{b (3 a^2+3 a b+b^2) \operatorname{Log}[b+a \operatorname{Cosh}[c+d x]^2]}{2 a^3 (a+b)^3 d} + \frac{\operatorname{Log}[\operatorname{Sinh}[c+d x]]}{(a+b)^3 d}$$

Result (type 3, 358 leaves):

$$\frac{1}{4 a^3 (a+b)^3 d (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2} (12 a^3 b^2+40 a^2 b^3+40 a b^4+12 b^5+9 a^4 b \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+33 a^3 b^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+51 a^2 b^3 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+32 a b^4 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+8 b^5 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+6 a^5 \operatorname{Log}[\operatorname{Sinh}[c+d x]]+16 a^4 b \operatorname{Log}[\operatorname{Sinh}[c+d x]]+16 a^3 b^2 \operatorname{Log}[\operatorname{Sinh}[c+d x]]+a^2 \operatorname{Cosh}[4(c+d x)](b(3 a^2+3 a b+b^2) \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+2 a^3 \operatorname{Log}[\operatorname{Sinh}[c+d x]])+4 a \operatorname{Cosh}[2(c+d x)](b^2(3 a^2+5 a b+2 b^2)+b(3 a^3+9 a^2 b+7 a b^2+2 b^3) \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+2 a^3(a+2 b) \operatorname{Log}[\operatorname{Sinh}[c+d x]]))$$

Problem 166: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c+d x]^2}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$\frac{x}{a^3} - \frac{b^{3/2} (35 a^2+28 a b+8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 a^3 (a+b)^{7/2} d} - \frac{(8 a^2-11 a b-4 b^2) \operatorname{Coth}[c+d x]}{8 a^2 (a+b)^3 d} - \frac{b \operatorname{Coth}[c+d x]}{4 a (a+b) d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} - \frac{b (9 a+4 b) \operatorname{Coth}[c+d x]}{8 a^2 (a+b)^2 d (a+b-b \operatorname{Tanh}[c+d x]^2)}$$

Result (type 3, 2083 leaves):

$$\frac{1}{(a+b)^3 (a+b \operatorname{Sech}[c+dx]^2)^3} (35a^2 + 28ab + 8b^2) (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6$$

$$\left(\left(i b^2 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \left(64a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) -$$

$$\left(i b^2 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / \left(64a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \Bigg) +$$

$$\frac{1}{512a^3 (a+b)^3 d (a+b \operatorname{Sech}[c+dx]^2)^3} (a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Csch}[c] \operatorname{Csch}[c+dx] \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^6$$

$$(8a^5 dx \operatorname{Cosh}[dx] + 56a^4 b dx \operatorname{Cosh}[dx] + 184a^3 b^2 dx \operatorname{Cosh}[dx] + 296a^2 b^3 dx \operatorname{Cosh}[dx] + 224a b^4 dx \operatorname{Cosh}[dx] + 64b^5 dx \operatorname{Cosh}[dx] -$$

$$12a^5 dx \operatorname{Cosh}[3dx] - 68a^4 b dx \operatorname{Cosh}[3dx] - 132a^3 b^2 dx \operatorname{Cosh}[3dx] - 108a^2 b^3 dx \operatorname{Cosh}[3dx] - 32a b^4 dx \operatorname{Cosh}[3dx] -$$

$$8a^5 dx \operatorname{Cosh}[2c-dx] - 56a^4 b dx \operatorname{Cosh}[2c-dx] - 184a^3 b^2 dx \operatorname{Cosh}[2c-dx] - 296a^2 b^3 dx \operatorname{Cosh}[2c-dx] - 224a b^4 dx \operatorname{Cosh}[2c-dx] -$$

$$64b^5 dx \operatorname{Cosh}[2c-dx] - 8a^5 dx \operatorname{Cosh}[2c+dx] - 56a^4 b dx \operatorname{Cosh}[2c+dx] - 184a^3 b^2 dx \operatorname{Cosh}[2c+dx] - 296a^2 b^3 dx \operatorname{Cosh}[2c+dx] -$$

$$224a b^4 dx \operatorname{Cosh}[2c+dx] - 64b^5 dx \operatorname{Cosh}[2c+dx] + 8a^5 dx \operatorname{Cosh}[4c+dx] + 56a^4 b dx \operatorname{Cosh}[4c+dx] + 184a^3 b^2 dx \operatorname{Cosh}[4c+dx] +$$

$$296a^2 b^3 dx \operatorname{Cosh}[4c+dx] + 224a b^4 dx \operatorname{Cosh}[4c+dx] + 64b^5 dx \operatorname{Cosh}[4c+dx] + 12a^5 dx \operatorname{Cosh}[2c+3dx] + 68a^4 b dx \operatorname{Cosh}[2c+3dx] +$$

$$132a^3 b^2 dx \operatorname{Cosh}[2c+3dx] + 108a^2 b^3 dx \operatorname{Cosh}[2c+3dx] + 32a b^4 dx \operatorname{Cosh}[2c+3dx] - 12a^5 dx \operatorname{Cosh}[4c+3dx] -$$

$$68a^4 b dx \operatorname{Cosh}[4c+3dx] - 132a^3 b^2 dx \operatorname{Cosh}[4c+3dx] - 108a^2 b^3 dx \operatorname{Cosh}[4c+3dx] - 32a b^4 dx \operatorname{Cosh}[4c+3dx] +$$

$$12a^5 dx \operatorname{Cosh}[6c+3dx] + 68a^4 b dx \operatorname{Cosh}[6c+3dx] + 132a^3 b^2 dx \operatorname{Cosh}[6c+3dx] + 108a^2 b^3 dx \operatorname{Cosh}[6c+3dx] +$$

$$32a b^4 dx \operatorname{Cosh}[6c+3dx] - 4a^5 dx \operatorname{Cosh}[2c+5dx] - 12a^4 b dx \operatorname{Cosh}[2c+5dx] - 12a^3 b^2 dx \operatorname{Cosh}[2c+5dx] - 4a^2 b^3 dx \operatorname{Cosh}[2c+5dx] +$$

$$4a^5 dx \operatorname{Cosh}[4c+5dx] + 12a^4 b dx \operatorname{Cosh}[4c+5dx] + 12a^3 b^2 dx \operatorname{Cosh}[4c+5dx] + 4a^2 b^3 dx \operatorname{Cosh}[4c+5dx] - 4a^5 dx \operatorname{Cosh}[6c+5dx] -$$

$$12a^4 b dx \operatorname{Cosh}[6c+5dx] - 12a^3 b^2 dx \operatorname{Cosh}[6c+5dx] - 4a^2 b^3 dx \operatorname{Cosh}[6c+5dx] + 4a^5 dx \operatorname{Cosh}[8c+5dx] + 12a^4 b dx \operatorname{Cosh}[8c+5dx] +$$

$$12a^3 b^2 dx \operatorname{Cosh}[8c+5dx] + 4a^2 b^3 dx \operatorname{Cosh}[8c+5dx] - 32a^5 \operatorname{Sinh}[dx] - 64a^4 b \operatorname{Sinh}[dx] - 30a^2 b^3 \operatorname{Sinh}[dx] - 120a b^4 \operatorname{Sinh}[dx] -$$

$$48b^5 \operatorname{Sinh}[dx] + 32a^5 \operatorname{Sinh}[3dx] + 64a^4 b \operatorname{Sinh}[3dx] + 26a^3 b^2 \operatorname{Sinh}[3dx] + 86a^2 b^3 \operatorname{Sinh}[3dx] + 32a b^4 \operatorname{Sinh}[3dx] - 48a^5 \operatorname{Sinh}[2c-dx] -$$

$$128a^4 b \operatorname{Sinh}[2c-dx] - 128a^3 b^2 \operatorname{Sinh}[2c-dx] - 30a^2 b^3 \operatorname{Sinh}[2c-dx] - 120a b^4 \operatorname{Sinh}[2c-dx] - 48b^5 \operatorname{Sinh}[2c-dx] +$$

$$48a^5 \operatorname{Sinh}[2c+dx] + 128a^4 b \operatorname{Sinh}[2c+dx] + 102a^3 b^2 \operatorname{Sinh}[2c+dx] - 86a^2 b^3 \operatorname{Sinh}[2c+dx] - 136a b^4 \operatorname{Sinh}[2c+dx] -$$

$$48b^5 \operatorname{Sinh}[2c+dx] - 32a^5 \operatorname{Sinh}[4c+dx] - 64a^4 b \operatorname{Sinh}[4c+dx] + 26a^3 b^2 \operatorname{Sinh}[4c+dx] + 86a^2 b^3 \operatorname{Sinh}[4c+dx] + 136a b^4 \operatorname{Sinh}[4c+dx] +$$

$$48b^5 \operatorname{Sinh}[4c+dx] - 8a^5 \operatorname{Sinh}[2c+3dx] - 26a^3 b^2 \operatorname{Sinh}[2c+3dx] - 86a^2 b^3 \operatorname{Sinh}[2c+3dx] - 32a b^4 \operatorname{Sinh}[2c+3dx] +$$

$$32a^5 \operatorname{Sinh}[4c+3dx] + 64a^4 b \operatorname{Sinh}[4c+3dx] - 13a^3 b^2 \operatorname{Sinh}[4c+3dx] - 36a^2 b^3 \operatorname{Sinh}[4c+3dx] - 16a b^4 \operatorname{Sinh}[4c+3dx] -$$

$$8a^5 \operatorname{Sinh}[6c+3dx] + 13a^3 b^2 \operatorname{Sinh}[6c+3dx] + 36a^2 b^3 \operatorname{Sinh}[6c+3dx] + 16a b^4 \operatorname{Sinh}[6c+3dx] + 8a^5 \operatorname{Sinh}[2c+5dx] +$$

$$13a^3 b^2 \operatorname{Sinh}[2c+5dx] + 6a^2 b^3 \operatorname{Sinh}[2c+5dx] - 13a^3 b^2 \operatorname{Sinh}[4c+5dx] - 6a^2 b^3 \operatorname{Sinh}[4c+5dx] + 8a^5 \operatorname{Sinh}[6c+5dx])$$

Problem 168: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c+dx]^4}{(a+b \operatorname{Sech}[c+dx]^2)^3} dx$$

Optimal (type 3, 232 leaves, 9 steps):

$$\frac{x}{a^3} \frac{b^{5/2} (63 a^2 + 36 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8 a^3 (a+b)^{9/2} d} - \frac{(8 a^3 + 32 a^2 b - 15 a b^2 - 4 b^3) \operatorname{Coth}[c+dx]}{8 a^2 (a+b)^4 d} -$$

$$\frac{(8 a^2 - 39 a b - 12 b^2) \operatorname{Coth}[c+dx]^3}{24 a^2 (a+b)^3 d} - \frac{b \operatorname{Coth}[c+dx]^3}{4 a (a+b) d (a+b - b \operatorname{Tanh}[c+dx]^2)^2} - \frac{b (11 a + 4 b) \operatorname{Coth}[c+dx]^3}{8 a^2 (a+b)^2 d (a+b - b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 3334 leaves):

$$\frac{1}{(a+b)^4 (a+b \operatorname{Sech}[c+dx]^2)^3} (63 a^2 + 36 a b + 8 b^2) (a + 2 b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c+dx]^6$$

$$\left(\left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2 b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \operatorname{Cosh}[2c] \right) / \left(64 a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \right.$$

$$\left. \left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2 b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \operatorname{Sinh}[2c] \right) / \left(64 a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \right) +$$

$$\frac{1}{6144 a^3 (a+b)^4 d (a+b \operatorname{Sech}[c+dx]^2)^3} (a + 2 b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Csch}[c] \operatorname{Csch}[c+dx]^3 \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^6$$

$$(-36 a^6 d x \operatorname{Cosh}[dx] - 336 a^5 b d x \operatorname{Cosh}[dx] - 1560 a^4 b^2 d x \operatorname{Cosh}[dx] - 3600 a^3 b^3 d x \operatorname{Cosh}[dx] - 4260 a^2 b^4 d x \operatorname{Cosh}[dx] - 2496 a b^5 d x \operatorname{Cosh}[dx] -$$

$$576 b^6 d x \operatorname{Cosh}[dx] + 36 a^6 d x \operatorname{Cosh}[3dx] + 240 a^5 b d x \operatorname{Cosh}[3dx] + 408 a^4 b^2 d x \operatorname{Cosh}[3dx] - 48 a^3 b^3 d x \operatorname{Cosh}[3dx] -$$

$$732 a^2 b^4 d x \operatorname{Cosh}[3dx] - 672 a b^5 d x \operatorname{Cosh}[3dx] - 192 b^6 d x \operatorname{Cosh}[3dx] + 36 a^6 d x \operatorname{Cosh}[2c - dx] + 336 a^5 b d x \operatorname{Cosh}[2c - dx] +$$

$$1560 a^4 b^2 d x \operatorname{Cosh}[2c - dx] + 3600 a^3 b^3 d x \operatorname{Cosh}[2c - dx] + 4260 a^2 b^4 d x \operatorname{Cosh}[2c - dx] + 2496 a b^5 d x \operatorname{Cosh}[2c - dx] +$$

$$576 b^6 d x \operatorname{Cosh}[2c - dx] + 36 a^6 d x \operatorname{Cosh}[2c + dx] + 336 a^5 b d x \operatorname{Cosh}[2c + dx] + 1560 a^4 b^2 d x \operatorname{Cosh}[2c + dx] + 3600 a^3 b^3 d x \operatorname{Cosh}[2c + dx] +$$

$$4260 a^2 b^4 d x \operatorname{Cosh}[2c + dx] + 2496 a b^5 d x \operatorname{Cosh}[2c + dx] + 576 b^6 d x \operatorname{Cosh}[2c + dx] - 36 a^6 d x \operatorname{Cosh}[4c + dx] - 336 a^5 b d x \operatorname{Cosh}[4c + dx] -$$

$$1560 a^4 b^2 d x \operatorname{Cosh}[4c + dx] - 3600 a^3 b^3 d x \operatorname{Cosh}[4c + dx] - 4260 a^2 b^4 d x \operatorname{Cosh}[4c + dx] - 2496 a b^5 d x \operatorname{Cosh}[4c + dx] -$$

$$576 b^6 d x \operatorname{Cosh}[4c + dx] - 36 a^6 d x \operatorname{Cosh}[2c + 3dx] - 240 a^5 b d x \operatorname{Cosh}[2c + 3dx] - 408 a^4 b^2 d x \operatorname{Cosh}[2c + 3dx] + 48 a^3 b^3 d x \operatorname{Cosh}[2c + 3dx] +$$

$$732 a^2 b^4 d x \operatorname{Cosh}[2c + 3dx] + 672 a b^5 d x \operatorname{Cosh}[2c + 3dx] + 192 b^6 d x \operatorname{Cosh}[2c + 3dx] + 36 a^6 d x \operatorname{Cosh}[4c + 3dx] +$$

$$240 a^5 b d x \operatorname{Cosh}[4c + 3dx] + 408 a^4 b^2 d x \operatorname{Cosh}[4c + 3dx] - 48 a^3 b^3 d x \operatorname{Cosh}[4c + 3dx] - 732 a^2 b^4 d x \operatorname{Cosh}[4c + 3dx] -$$

$$672 a b^5 d x \operatorname{Cosh}[4c + 3dx] - 192 b^6 d x \operatorname{Cosh}[4c + 3dx] - 36 a^6 d x \operatorname{Cosh}[6c + 3dx] - 240 a^5 b d x \operatorname{Cosh}[6c + 3dx] -$$

$$408 a^4 b^2 d x \operatorname{Cosh}[6c + 3dx] + 48 a^3 b^3 d x \operatorname{Cosh}[6c + 3dx] + 732 a^2 b^4 d x \operatorname{Cosh}[6c + 3dx] + 672 a b^5 d x \operatorname{Cosh}[6c + 3dx] +$$

$$192 b^6 d x \operatorname{Cosh}[6c + 3dx] - 12 a^6 d x \operatorname{Cosh}[2c + 5dx] - 144 a^5 b d x \operatorname{Cosh}[2c + 5dx] - 456 a^4 b^2 d x \operatorname{Cosh}[2c + 5dx] -$$

$$624 a^3 b^3 d x \operatorname{Cosh}[2c + 5dx] - 396 a^2 b^4 d x \operatorname{Cosh}[2c + 5dx] - 96 a b^5 d x \operatorname{Cosh}[2c + 5dx] + 12 a^6 d x \operatorname{Cosh}[4c + 5dx] +$$

$$144 a^5 b d x \operatorname{Cosh}[4c + 5dx] + 456 a^4 b^2 d x \operatorname{Cosh}[4c + 5dx] + 624 a^3 b^3 d x \operatorname{Cosh}[4c + 5dx] + 396 a^2 b^4 d x \operatorname{Cosh}[4c + 5dx] +$$

$$96 a b^5 d x \operatorname{Cosh}[4c + 5dx] - 12 a^6 d x \operatorname{Cosh}[6c + 5dx] - 144 a^5 b d x \operatorname{Cosh}[6c + 5dx] - 456 a^4 b^2 d x \operatorname{Cosh}[6c + 5dx] -$$

$$624 a^3 b^3 d x \operatorname{Cosh}[6c + 5dx] - 396 a^2 b^4 d x \operatorname{Cosh}[6c + 5dx] - 96 a b^5 d x \operatorname{Cosh}[6c + 5dx] + 12 a^6 d x \operatorname{Cosh}[8c + 5dx] +$$

$$144 a^5 b d x \operatorname{Cosh}[8c + 5dx] + 456 a^4 b^2 d x \operatorname{Cosh}[8c + 5dx] + 624 a^3 b^3 d x \operatorname{Cosh}[8c + 5dx] + 396 a^2 b^4 d x \operatorname{Cosh}[8c + 5dx] +$$

$$\begin{aligned}
& 96 a b^5 d x \operatorname{Cosh}[8 c + 5 d x] - 12 a^6 d x \operatorname{Cosh}[4 c + 7 d x] - 48 a^5 b d x \operatorname{Cosh}[4 c + 7 d x] - 72 a^4 b^2 d x \operatorname{Cosh}[4 c + 7 d x] - 48 a^3 b^3 d x \operatorname{Cosh}[4 c + 7 d x] - \\
& 12 a^2 b^4 d x \operatorname{Cosh}[4 c + 7 d x] + 12 a^6 d x \operatorname{Cosh}[6 c + 7 d x] + 48 a^5 b d x \operatorname{Cosh}[6 c + 7 d x] + 72 a^4 b^2 d x \operatorname{Cosh}[6 c + 7 d x] + \\
& 48 a^3 b^3 d x \operatorname{Cosh}[6 c + 7 d x] + 12 a^2 b^4 d x \operatorname{Cosh}[6 c + 7 d x] - 12 a^6 d x \operatorname{Cosh}[8 c + 7 d x] - 48 a^5 b d x \operatorname{Cosh}[8 c + 7 d x] - \\
& 72 a^4 b^2 d x \operatorname{Cosh}[8 c + 7 d x] - 48 a^3 b^3 d x \operatorname{Cosh}[8 c + 7 d x] - 12 a^2 b^4 d x \operatorname{Cosh}[8 c + 7 d x] + 12 a^6 d x \operatorname{Cosh}[10 c + 7 d x] + \\
& 48 a^5 b d x \operatorname{Cosh}[10 c + 7 d x] + 72 a^4 b^2 d x \operatorname{Cosh}[10 c + 7 d x] + 48 a^3 b^3 d x \operatorname{Cosh}[10 c + 7 d x] + 12 a^2 b^4 d x \operatorname{Cosh}[10 c + 7 d x] - 128 a^6 \operatorname{Sinh}[d x] - \\
& 440 a^5 b \operatorname{Sinh}[d x] - 1152 a^4 b^2 \operatorname{Sinh}[d x] - 1920 a^3 b^3 \operatorname{Sinh}[d x] + 228 a^2 b^4 \operatorname{Sinh}[d x] + 1320 a b^5 \operatorname{Sinh}[d x] + 432 b^6 \operatorname{Sinh}[d x] + \\
& 48 a^6 \operatorname{Sinh}[3 d x] + 104 a^5 b \operatorname{Sinh}[3 d x] + 640 a^4 b^2 \operatorname{Sinh}[3 d x] + 1511 a^3 b^3 \operatorname{Sinh}[3 d x] - 528 a^2 b^4 \operatorname{Sinh}[3 d x] + 264 a b^5 \operatorname{Sinh}[3 d x] + \\
& 144 b^6 \operatorname{Sinh}[3 d x] - 32 a^6 \operatorname{Sinh}[2 c - d x] + 384 a^5 b \operatorname{Sinh}[2 c - d x] + 2048 a^4 b^2 \operatorname{Sinh}[2 c - d x] + 3072 a^3 b^3 \operatorname{Sinh}[2 c - d x] + \\
& 228 a^2 b^4 \operatorname{Sinh}[2 c - d x] + 1320 a b^5 \operatorname{Sinh}[2 c - d x] + 432 b^6 \operatorname{Sinh}[2 c - d x] + 32 a^6 \operatorname{Sinh}[2 c + d x] - 384 a^5 b \operatorname{Sinh}[2 c + d x] - \\
& 2048 a^4 b^2 \operatorname{Sinh}[2 c + d x] - 2919 a^3 b^3 \operatorname{Sinh}[2 c + d x] + 642 a^2 b^4 \operatorname{Sinh}[2 c + d x] + 1416 a b^5 \operatorname{Sinh}[2 c + d x] + 432 b^6 \operatorname{Sinh}[2 c + d x] - \\
& 128 a^6 \operatorname{Sinh}[4 c + d x] - 440 a^5 b \operatorname{Sinh}[4 c + d x] - 1152 a^4 b^2 \operatorname{Sinh}[4 c + d x] - 2073 a^3 b^3 \operatorname{Sinh}[4 c + d x] - 642 a^2 b^4 \operatorname{Sinh}[4 c + d x] - \\
& 1416 a b^5 \operatorname{Sinh}[4 c + d x] - 432 b^6 \operatorname{Sinh}[4 c + d x] - 144 a^6 \operatorname{Sinh}[2 c + 3 d x] - 672 a^5 b \operatorname{Sinh}[2 c + 3 d x] - 960 a^4 b^2 \operatorname{Sinh}[2 c + 3 d x] + \\
& 153 a^3 b^3 \operatorname{Sinh}[2 c + 3 d x] + 528 a^2 b^4 \operatorname{Sinh}[2 c + 3 d x] - 264 a b^5 \operatorname{Sinh}[2 c + 3 d x] - 144 b^6 \operatorname{Sinh}[2 c + 3 d x] + 48 a^6 \operatorname{Sinh}[4 c + 3 d x] + \\
& 104 a^5 b \operatorname{Sinh}[4 c + 3 d x] + 640 a^4 b^2 \operatorname{Sinh}[4 c + 3 d x] + 1664 a^3 b^3 \operatorname{Sinh}[4 c + 3 d x] - 66 a^2 b^4 \operatorname{Sinh}[4 c + 3 d x] - 408 a b^5 \operatorname{Sinh}[4 c + 3 d x] - \\
& 144 b^6 \operatorname{Sinh}[4 c + 3 d x] - 144 a^6 \operatorname{Sinh}[6 c + 3 d x] - 672 a^5 b \operatorname{Sinh}[6 c + 3 d x] - 960 a^4 b^2 \operatorname{Sinh}[6 c + 3 d x] + 66 a^2 b^4 \operatorname{Sinh}[6 c + 3 d x] + \\
& 408 a b^5 \operatorname{Sinh}[6 c + 3 d x] + 144 b^6 \operatorname{Sinh}[6 c + 3 d x] + 80 a^6 \operatorname{Sinh}[2 c + 5 d x] + 480 a^5 b \operatorname{Sinh}[2 c + 5 d x] + 832 a^4 b^2 \operatorname{Sinh}[2 c + 5 d x] + \\
& 294 a^2 b^4 \operatorname{Sinh}[2 c + 5 d x] + 96 a b^5 \operatorname{Sinh}[2 c + 5 d x] - 48 a^6 \operatorname{Sinh}[4 c + 5 d x] - 120 a^5 b \operatorname{Sinh}[4 c + 5 d x] - 294 a^2 b^4 \operatorname{Sinh}[4 c + 5 d x] - \\
& 96 a b^5 \operatorname{Sinh}[4 c + 5 d x] + 80 a^6 \operatorname{Sinh}[6 c + 5 d x] + 480 a^5 b \operatorname{Sinh}[6 c + 5 d x] + 832 a^4 b^2 \operatorname{Sinh}[6 c + 5 d x] - 51 a^3 b^3 \operatorname{Sinh}[6 c + 5 d x] - \\
& 132 a^2 b^4 \operatorname{Sinh}[6 c + 5 d x] - 48 a b^5 \operatorname{Sinh}[6 c + 5 d x] - 48 a^6 \operatorname{Sinh}[8 c + 5 d x] - 120 a^5 b \operatorname{Sinh}[8 c + 5 d x] + 51 a^3 b^3 \operatorname{Sinh}[8 c + 5 d x] + \\
& 132 a^2 b^4 \operatorname{Sinh}[8 c + 5 d x] + 48 a b^5 \operatorname{Sinh}[8 c + 5 d x] + 32 a^6 \operatorname{Sinh}[4 c + 7 d x] + 104 a^5 b \operatorname{Sinh}[4 c + 7 d x] + 51 a^3 b^3 \operatorname{Sinh}[4 c + 7 d x] + \\
& 18 a^2 b^4 \operatorname{Sinh}[4 c + 7 d x] - 51 a^3 b^3 \operatorname{Sinh}[6 c + 7 d x] - 18 a^2 b^4 \operatorname{Sinh}[6 c + 7 d x] + 32 a^6 \operatorname{Sinh}[8 c + 7 d x] + 104 a^5 b \operatorname{Sinh}[8 c + 7 d x]
\end{aligned}$$

Problem 169: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Sech}[c + d x]^2)^4} dx$$

Optimal (type 3, 207 leaves, 7 steps):

$$\begin{aligned}
& \frac{x}{a^4} - \frac{\sqrt{b} (35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{16 a^4 (a + b)^{7/2} d} - \frac{b \operatorname{Tanh}[c + d x]}{6 a (a + b) d (a + b - b \operatorname{Tanh}[c + d x]^2)^3} \\
& - \frac{b (11 a + 6 b) \operatorname{Tanh}[c + d x]}{24 a^2 (a + b)^2 d (a + b - b \operatorname{Tanh}[c + d x]^2)^2} - \frac{b (19 a^2 + 22 a b + 8 b^2) \operatorname{Tanh}[c + d x]}{16 a^3 (a + b)^3 d (a + b - b \operatorname{Tanh}[c + d x]^2)}
\end{aligned}$$

Result (type 3, 1405 leaves):

$$\frac{1}{(a+b)^3 (a+b \operatorname{Sech}[c+dx])^4} (35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3) (a+2b+a \operatorname{Cosh}[2c+2dx])^4$$

$$\operatorname{Sech}[c+dx]^8 \left(\left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \left(256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) -$$

$$\left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / \left(256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \Bigg) +$$

$$\frac{1}{3072 a^4 (a+b)^3 d (a+b \operatorname{Sech}[c+dx])^4} (a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^8$$

$$(480 a^6 d x \operatorname{Cosh}[2c] + 3168 a^5 b d x \operatorname{Cosh}[2c] + 8928 a^4 b^2 d x \operatorname{Cosh}[2c] + 14112 a^3 b^3 d x \operatorname{Cosh}[2c] + 13248 a^2 b^4 d x \operatorname{Cosh}[2c] +$$

$$6912 a b^5 d x \operatorname{Cosh}[2c] + 1536 b^6 d x \operatorname{Cosh}[2c] + 360 a^6 d x \operatorname{Cosh}[2dx] + 2232 a^5 b d x \operatorname{Cosh}[2dx] + 5688 a^4 b^2 d x \operatorname{Cosh}[2dx] +$$

$$7272 a^3 b^3 d x \operatorname{Cosh}[2dx] + 4608 a^2 b^4 d x \operatorname{Cosh}[2dx] + 1152 a b^5 d x \operatorname{Cosh}[2dx] + 360 a^6 d x \operatorname{Cosh}[4c+2dx] +$$

$$2232 a^5 b d x \operatorname{Cosh}[4c+2dx] + 5688 a^4 b^2 d x \operatorname{Cosh}[4c+2dx] + 7272 a^3 b^3 d x \operatorname{Cosh}[4c+2dx] + 4608 a^2 b^4 d x \operatorname{Cosh}[4c+2dx] +$$

$$1152 a b^5 d x \operatorname{Cosh}[4c+2dx] + 144 a^6 d x \operatorname{Cosh}[2c+4dx] + 720 a^5 b d x \operatorname{Cosh}[2c+4dx] + 1296 a^4 b^2 d x \operatorname{Cosh}[2c+4dx] +$$

$$1008 a^3 b^3 d x \operatorname{Cosh}[2c+4dx] + 288 a^2 b^4 d x \operatorname{Cosh}[2c+4dx] + 144 a^6 d x \operatorname{Cosh}[6c+4dx] + 720 a^5 b d x \operatorname{Cosh}[6c+4dx] +$$

$$1296 a^4 b^2 d x \operatorname{Cosh}[6c+4dx] + 1008 a^3 b^3 d x \operatorname{Cosh}[6c+4dx] + 288 a^2 b^4 d x \operatorname{Cosh}[6c+4dx] + 24 a^6 d x \operatorname{Cosh}[4c+6dx] +$$

$$72 a^5 b d x \operatorname{Cosh}[4c+6dx] + 72 a^4 b^2 d x \operatorname{Cosh}[4c+6dx] + 24 a^3 b^3 d x \operatorname{Cosh}[4c+6dx] + 24 a^6 d x \operatorname{Cosh}[8c+6dx] +$$

$$72 a^5 b d x \operatorname{Cosh}[8c+6dx] + 72 a^4 b^2 d x \operatorname{Cosh}[8c+6dx] + 24 a^3 b^3 d x \operatorname{Cosh}[8c+6dx] + 870 a^5 b \operatorname{Sinh}[2c] + 4292 a^4 b^2 \operatorname{Sinh}[2c] +$$

$$8792 a^3 b^3 \operatorname{Sinh}[2c] + 9936 a^2 b^4 \operatorname{Sinh}[2c] + 5824 a b^5 \operatorname{Sinh}[2c] + 1408 b^6 \operatorname{Sinh}[2c] - 870 a^5 b \operatorname{Sinh}[2dx] - 3792 a^4 b^2 \operatorname{Sinh}[2dx] -$$

$$6432 a^3 b^3 \operatorname{Sinh}[2dx] - 4608 a^2 b^4 \operatorname{Sinh}[2dx] - 1248 a b^5 \operatorname{Sinh}[2dx] + 435 a^5 b \operatorname{Sinh}[4c+2dx] + 2124 a^4 b^2 \operatorname{Sinh}[4c+2dx] +$$

$$3972 a^3 b^3 \operatorname{Sinh}[4c+2dx] + 3072 a^2 b^4 \operatorname{Sinh}[4c+2dx] + 864 a b^5 \operatorname{Sinh}[4c+2dx] - 435 a^5 b \operatorname{Sinh}[2c+4dx] -$$

$$1374 a^4 b^2 \operatorname{Sinh}[2c+4dx] - 1248 a^3 b^3 \operatorname{Sinh}[2c+4dx] - 384 a^2 b^4 \operatorname{Sinh}[2c+4dx] + 87 a^5 b \operatorname{Sinh}[6c+4dx] + 366 a^4 b^2 \operatorname{Sinh}[6c+4dx] +$$

$$408 a^3 b^3 \operatorname{Sinh}[6c+4dx] + 144 a^2 b^4 \operatorname{Sinh}[6c+4dx] - 87 a^5 b \operatorname{Sinh}[4c+6dx] - 116 a^4 b^2 \operatorname{Sinh}[4c+6dx] - 44 a^3 b^3 \operatorname{Sinh}[4c+6dx])$$

Problem 181: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+b \operatorname{Sech}[x]^2} dx$$

Optimal (type 3, 59 leaves, 6 steps):

$$\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]}{\sqrt{a+b-b \operatorname{Tanh}[x]^2}}\right] + \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tanh}[x]}{\sqrt{a+b-b \operatorname{Tanh}[x]^2}}\right]$$

Result (type 3, 134 leaves):

$$\frac{1}{a + 2b + a \operatorname{Cosh}[2x]} \sqrt{2} \operatorname{Cosh}[x] \left(\sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{b} \operatorname{Sinh}[x]}{\sqrt{a + 2b + a \operatorname{Cosh}[2x]}} \right] \sqrt{a + 2b + a \operatorname{Cosh}[2x]} + \sqrt{a} \sqrt{a + b} \operatorname{ArcSinh} \left[\frac{\sqrt{a} \operatorname{Sinh}[x]}{\sqrt{a + b}} \right] \sqrt{\frac{a + 2b + a \operatorname{Cosh}[2x]}{a + b}} \right) \sqrt{a + b \operatorname{Sech}[x]^2}$$

Problem 189: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[x]^2)^{3/2} \operatorname{Tanh}[x] dx$$

Optimal (type 3, 57 leaves, 6 steps):

$$a^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a}} \right] - a \sqrt{a + b \operatorname{Sech}[x]^2} - \frac{1}{3} (a + b \operatorname{Sech}[x]^2)^{3/2}$$

Result (type 3, 117 leaves):

$$- \left(\left(2 \left(b \sqrt{a + 2b + a \operatorname{Cosh}[2x]} + 4a \operatorname{Cosh}[x]^2 \sqrt{a + 2b + a \operatorname{Cosh}[2x]} - 3 \sqrt{2} a^{3/2} \operatorname{Cosh}[x]^3 \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2b + a \operatorname{Cosh}[2x]} \right] \right) \right) (a + b \operatorname{Sech}[x]^2)^{3/2} \right) / \left(3 (a + 2b + a \operatorname{Cosh}[2x])^{3/2} \right)$$

Problem 191: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[x] (a + b \operatorname{Sech}[x]^2)^{3/2} dx$$

Optimal (type 3, 70 leaves, 8 steps):

$$a^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a}} \right] - (a + b)^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a + b}} \right] + b \sqrt{a + b \operatorname{Sech}[x]^2}$$

Result (type 3, 159 leaves):

$$- \left(\left(2 (b + a \operatorname{Cosh}[x]^2) \left(\sqrt{2} (a + b)^2 \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a + b} \operatorname{Cosh}[x]}{\sqrt{a + 2b + a \operatorname{Cosh}[2x]}} \right] \operatorname{Cosh}[x] - \sqrt{a + b} \left(b \sqrt{a + 2b + a \operatorname{Cosh}[2x]} + \sqrt{2} a^{3/2} \operatorname{Cosh}[x] \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2b + a \operatorname{Cosh}[2x]} \right] \right) \right) \right) \sqrt{a + b \operatorname{Sech}[x]^2} \right) / \left(\sqrt{a + b} (a + 2b + a \operatorname{Cosh}[2x])^{3/2} \right)$$

Problem 196: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^3}{\sqrt{a + b \text{Sech}[x]^2}} dx$$

Optimal (type 3, 42 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}} + \frac{\sqrt{a + b \text{Sech}[x]^2}}{b}$$

Result (type 3, 105 leaves):

$$\frac{\sqrt{a + 2b + a \text{Cosh}[2x]} \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cosh}[x] + \sqrt{a + 2b + a \text{Cosh}[2x]}\right] \text{Sech}[x]}{\sqrt{2} \sqrt{a} \sqrt{a + b \text{Sech}[x]^2}} + \frac{(a + 2b + a \text{Cosh}[2x]) \text{Sech}[x]^2}{2b \sqrt{a + b \text{Sech}[x]^2}}$$

Problem 198: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{\sqrt{a + b \text{Sech}[x]^2}} dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 70 leaves):

$$\frac{\sqrt{a + 2b + a \text{Cosh}[2x]} \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cosh}[x] + \sqrt{a + 2b + a \text{Cosh}[2x]}\right] \text{Sech}[x]}{\sqrt{2} \sqrt{a} \sqrt{a + b \text{Sech}[x]^2}}$$

Problem 199: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b \text{Sech}[x]^2}} dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a} \text{Tanh}[x]}{\sqrt{a+b \text{Tanh}[x]^2}}\right]}{\sqrt{a}}$$

Result (type 3, 69 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \text{Sinh}[x]}{\sqrt{a+2b+a \text{Cosh}[2x]}}\right] \sqrt{a+2b+a \text{Cosh}[2x]} \text{Sech}[x]}{\sqrt{2} \sqrt{a} \sqrt{a+b \text{Sech}[x]^2}}$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b \text{Sech}[x]^2}} dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 124 leaves):

$$\left(\sqrt{a+2b+a \text{Cosh}[2x]} \left(-\sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a+b} \text{Cosh}[x]}{\sqrt{a+2b+a \text{Cosh}[2x]}}\right] + \sqrt{a+b} \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cosh}[x] + \sqrt{a+2b+a \text{Cosh}[2x]}\right] \right) \text{Sech}[x] \right) / \left(\sqrt{2} \sqrt{a} \sqrt{a+b} \sqrt{a+b \text{Sech}[x]^2} \right)$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^3}{(a+b \text{Sech}[x]^2)^{3/2}} dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{a+b}{a b \sqrt{a+b \text{Sech}[x]^2}}$$

Result (type 3, 103 leaves):

$$\frac{1}{4 a^{3/2} (a + b \operatorname{Sech}[x]^2)^{3/2}} \left(-\frac{2 \sqrt{a} (a + b) \operatorname{Cosh}[x] (a + 2 b + a \operatorname{Cosh}[2 x])}{b} + \sqrt{2} (a + 2 b + a \operatorname{Cosh}[2 x])^{3/2} \operatorname{Log}[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2 b + a \operatorname{Cosh}[2 x]}] \right) \operatorname{Sech}[x]^3$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^2}{(a + b \operatorname{Sech}[x]^2)^{3/2}} dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^2}}\right]}{a^{3/2}} - \frac{\operatorname{Tanh}[x]}{a \sqrt{a + b - b \operatorname{Tanh}[x]^2}}$$

Result (type 3, 105 leaves):

$$-\frac{1}{4 a^{3/2} (a + b \operatorname{Sech}[x]^2)^{3/2}} \operatorname{Sech}[x]^3 \left(-\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \operatorname{Sinh}[x]}{\sqrt{a + 2 b + a \operatorname{Cosh}[2 x]}}\right] (a + 2 b + a \operatorname{Cosh}[2 x])^{3/2} + a^{3/2} \operatorname{Sinh}[x] + 4 \sqrt{a} b \operatorname{Sinh}[x] + a^{3/2} \operatorname{Sinh}[3 x] \right)$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]}{(a + b \operatorname{Sech}[x]^2)^{3/2}} dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{1}{a \sqrt{a + b \operatorname{Sech}[x]^2}}$$

Result (type 3, 98 leaves):

$$-\frac{1}{4 a^{3/2} (a + b \operatorname{Sech}[x]^2)^{3/2}} (a + 2 b + a \operatorname{Cosh}[2 x]) \left(2 \sqrt{a} \operatorname{Cosh}[x] - \sqrt{2} \sqrt{a + 2 b + a \operatorname{Cosh}[2 x]} \operatorname{Log}[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2 b + a \operatorname{Cosh}[2 x]}] \right) \operatorname{Sech}[x]^3$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{(a + b \text{Sech}[x]^2)^{5/2}} dx$$

Optimal (type 3, 109 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a}}\right]}{a^{5/2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{b}{3 a (a+b) (a+b \text{Sech}[x]^2)^{3/2}} - \frac{b (2 a+b)}{a^2 (a+b)^2 \sqrt{a+b \text{Sech}[x]^2}}$$

Result (type 3, 242 leaves):

$$\frac{1}{8 (a+b \text{Sech}[x]^2)^{5/2}} \left(-\frac{2 b \text{Cosh}[x] (a+2 b+a \text{Cosh}[2 x]) (7 a^2+16 a b+6 b^2+a (7 a+4 b) \text{Cosh}[2 x])}{3 a^2 (a+b)^2} - \frac{1}{\sqrt{2} a^{5/2} (a+b)^{5/2}} (a+2 b+a \text{Cosh}[2 x])^{5/2} \left(\sqrt{a} (a^2-2 a b-b^2) \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a+b} \text{Cosh}[x]}{\sqrt{a+2 b+a \text{Cosh}[2 x]}}\right] + (a+b)^2 \left(\sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{2 a+2 b} \text{Cosh}[x]}{\sqrt{a+2 b+a \text{Cosh}[2 x]}}\right] - 2 \sqrt{a+b} \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cosh}[x] + \sqrt{a+2 b+a \text{Cosh}[2 x]}\right] \right) \right) \right) \text{Sech}[x]^5$$

Problem 220: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b \text{Sech}[c+d x]^2)^{7/2}} dx$$

Optimal (type 3, 183 leaves, 7 steps):

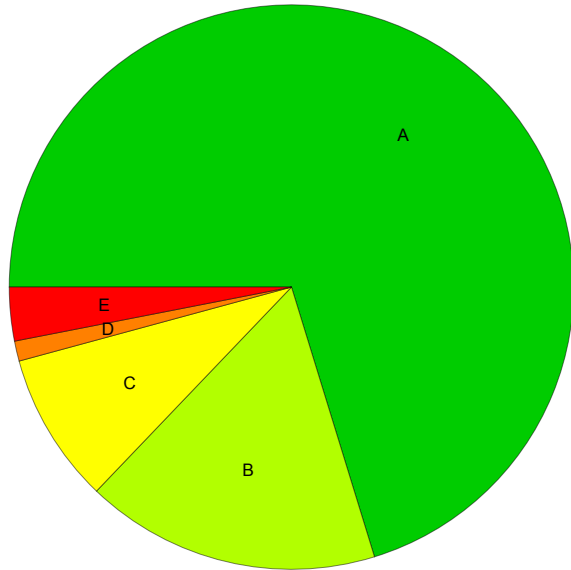
$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a} \text{Tanh}[c+d x]}{\sqrt{a+b-b \text{Tanh}[c+d x]^2}}\right]}{a^{7/2} d} - \frac{b \text{Tanh}[c+d x]}{5 a (a+b) d (a+b-b \text{Tanh}[c+d x]^2)^{5/2}} - \frac{b (9 a+5 b) \text{Tanh}[c+d x]}{15 a^2 (a+b)^2 d (a+b-b \text{Tanh}[c+d x]^2)^{3/2}} - \frac{b (33 a^2+40 a b+15 b^2) \text{Tanh}[c+d x]}{15 a^3 (a+b)^3 d \sqrt{a+b-b \text{Tanh}[c+d x]^2}}$$

Result (type 3, 749 leaves):

$$\begin{aligned}
& \frac{1}{8 a^3 (a + b \operatorname{Sech}[c + d x]^2)^{7/2}} (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^{7/2} \\
& \left(\left(e^{-3 c - d x} (1 + e^{2 c}) \sqrt{4 b e^{2(c+d x)} + a (1 + e^{2(c+d x)})^2} \left(\operatorname{Log}\left[e^{-2 c} \left(a + 2 b + a e^{2(c+d x)} + \sqrt{a} \sqrt{4 b e^{2(c+d x)} + a (1 + e^{2(c+d x)})^2} \right) \right] + e^{2 c} \left(2 d x - \operatorname{Log}\left[\right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. e^{-2 c} \left(a + a e^{2(c+d x)} + 2 b e^{2(c+d x)} + \sqrt{a} \sqrt{4 b e^{2(c+d x)} + a (1 + e^{2(c+d x)})^2} \right) \right] \right) \right) \right) / \left(4 \sqrt{2} \sqrt{a} d \sqrt{4 b + a e^{-2(c+d x)} (1 + e^{2(c+d x)})^2} \right) + \\
& \left(e^{-3 c - d x} (-1 + e^{2 c}) \sqrt{4 b e^{2(c+d x)} + a (1 + e^{2(c+d x)})^2} \left(\operatorname{Log}\left[e^{-2 c} \left(a + 2 b + a e^{2(c+d x)} + \sqrt{a} \sqrt{4 b e^{2(c+d x)} + a (1 + e^{2(c+d x)})^2} \right) \right] \right) + \right. \\
& \quad \left. e^{2 c} \left(-2 d x + \operatorname{Log}\left[e^{-2 c} \left(a + a e^{2(c+d x)} + 2 b e^{2(c+d x)} + \sqrt{a} \sqrt{4 b e^{2(c+d x)} + a (1 + e^{2(c+d x)})^2} \right) \right] \right) \right) / \\
& \left(4 \sqrt{2} \sqrt{a} d \sqrt{4 b + a e^{-2(c+d x)} (1 + e^{2(c+d x)})^2} \right) \left) \operatorname{Sech}[c + d x]^7 + \frac{1}{(a + b \operatorname{Sech}[c + d x]^2)^{7/2}} \right. \\
& (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^4 \operatorname{Sech}[c + d x]^7 \left(-\frac{b^3 \operatorname{Sinh}[c + d x]}{10 a^3 (a + b) d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3} + \right. \\
& \quad \left. \frac{-45 a^2 b \operatorname{Sinh}[c + d x] - 60 a b^2 \operatorname{Sinh}[c + d x] - 23 b^3 \operatorname{Sinh}[c + d x]}{120 a^3 (a + b)^3 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])} + \right. \\
& \quad \left. \frac{15 a b^2 \operatorname{Sinh}[c + d x] + 11 b^3 \operatorname{Sinh}[c + d x]}{60 a^3 (a + b)^2 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2} \right)
\end{aligned}$$

Summary of Integration Test Results

521 integration problems



A - 366 optimal antiderivatives

B - 88 more than twice size of optimal antiderivatives

C - 45 unnecessarily complex antiderivatives

D - 6 unable to integrate problems

E - 16 integration timeouts